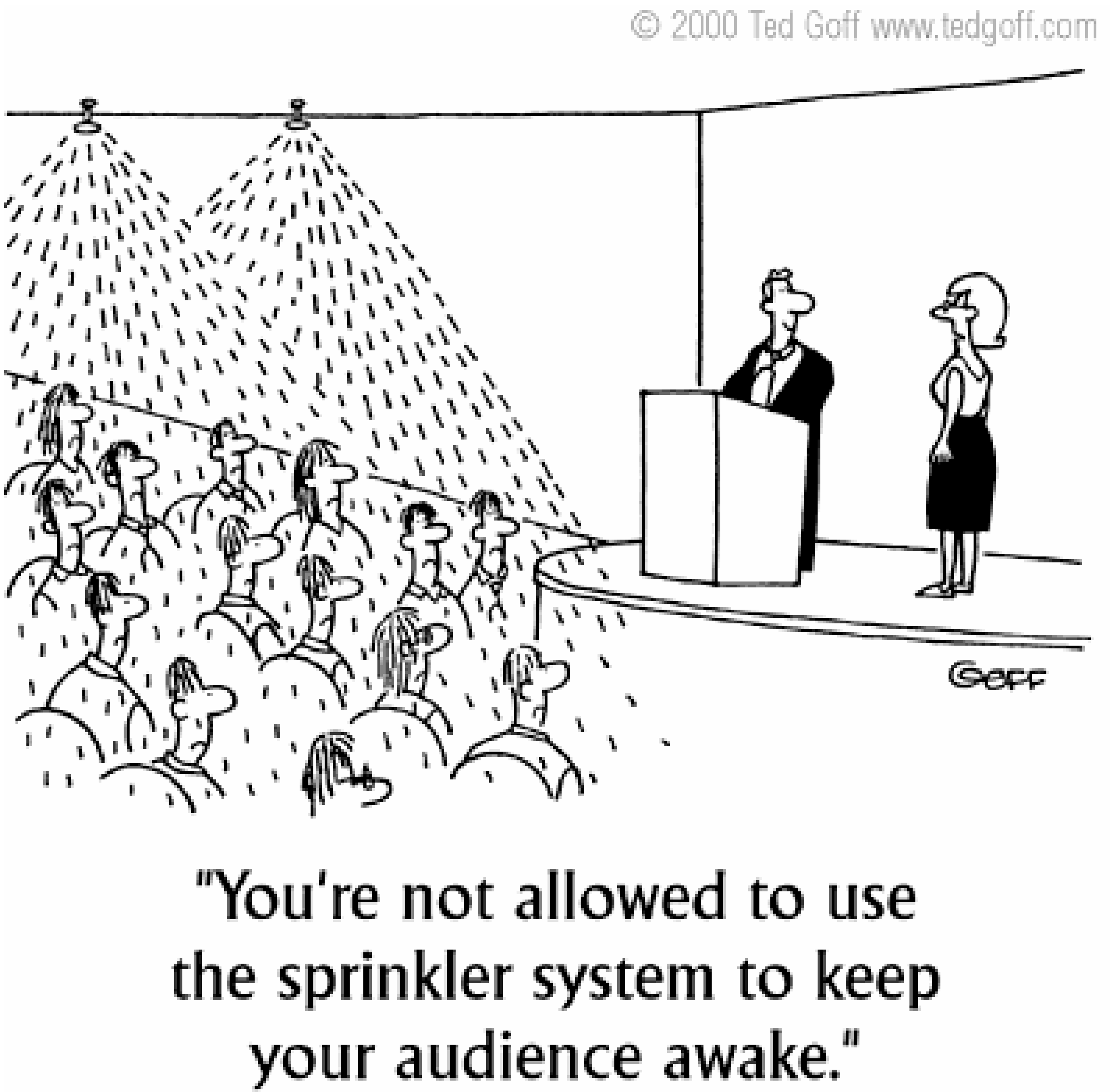
Social Network Analysis

Basic Concepts, Methods & Theory



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GOFF
"You're not allowed to use the sprinkler system to keep your audience awake."

Textbooks

- Gloor (2006): *Swarm Creativity – Competitive Advantage Through Collaborative Innovation Networks*, Oxford: Oxford University Press.
- Gloor & Cooper (2007): *Coolhunting – Chasing Down the Next Big Thing*, New York: Mcgraw-Hill Professional.
- Hanneman & Riddle (2005) *Introduction to Social Network Methods,* available at *http*://faculty.ucr.edu/~hanneman/nettext/
- Wasserman & Faust (1994): *Social Network Analysis – Methods and Applications*, Cambridge: Cambridge University Press.

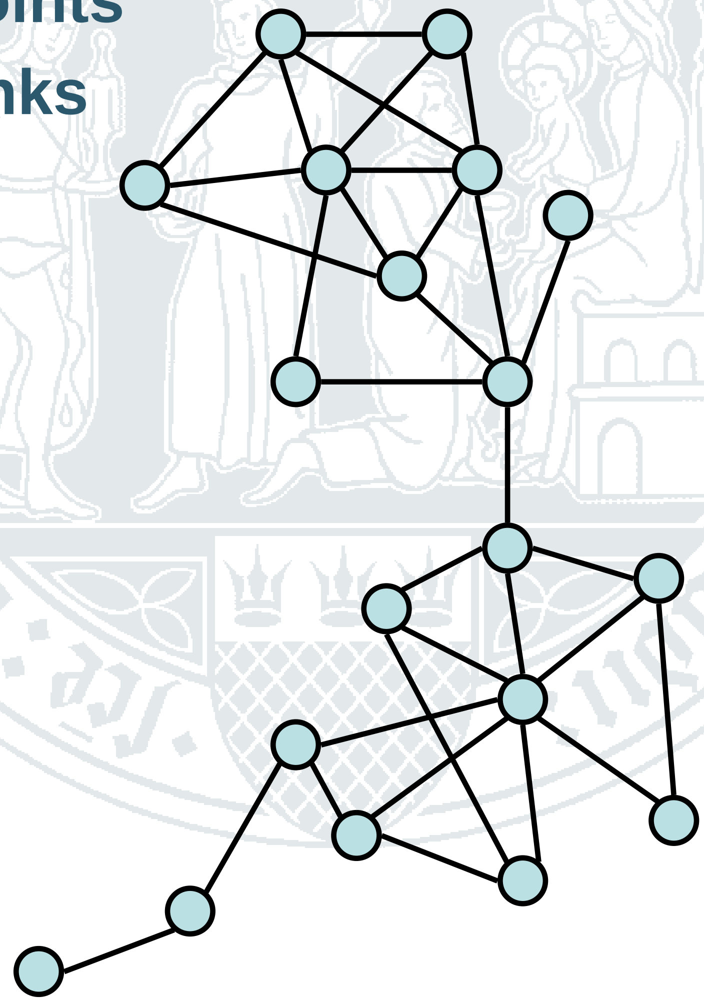
Introduction

Basic Concepts

What is a network?

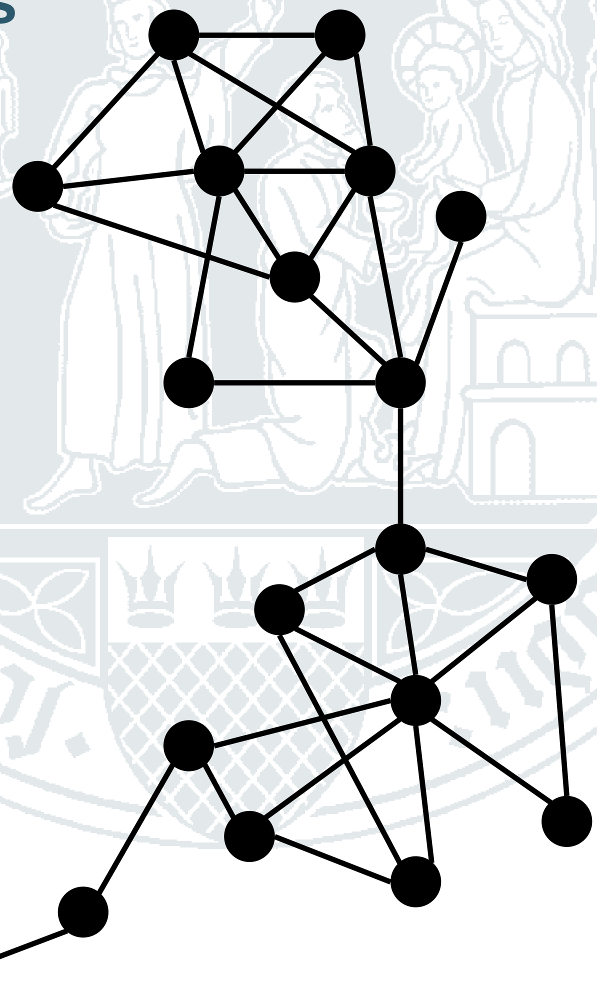
What is a Network?

- Actors / nodes / vertices / points
- Ties / edges / arcs / lines / links



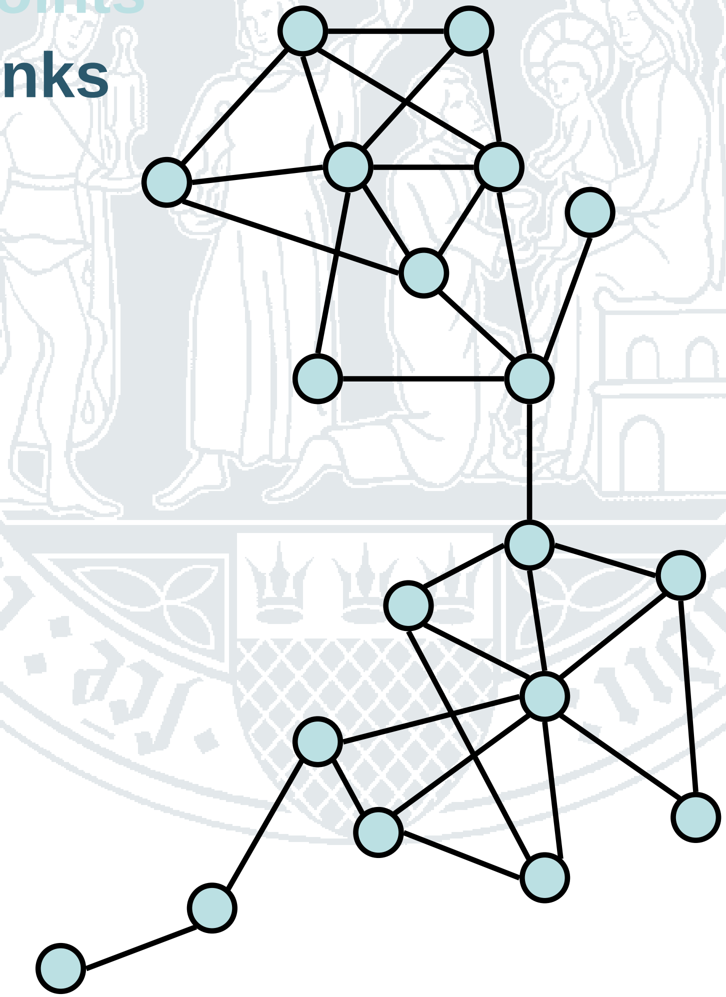
What is a Network?

- **Actors / nodes / vertices / points**
  - **Computers / Telephones**
  - **Persons / Employees**
  - **Companies / Business Units**
  - **Articles / Books**
  - **Can have properties (attributes)**
- **Ties / edges / arcs / lines / links**

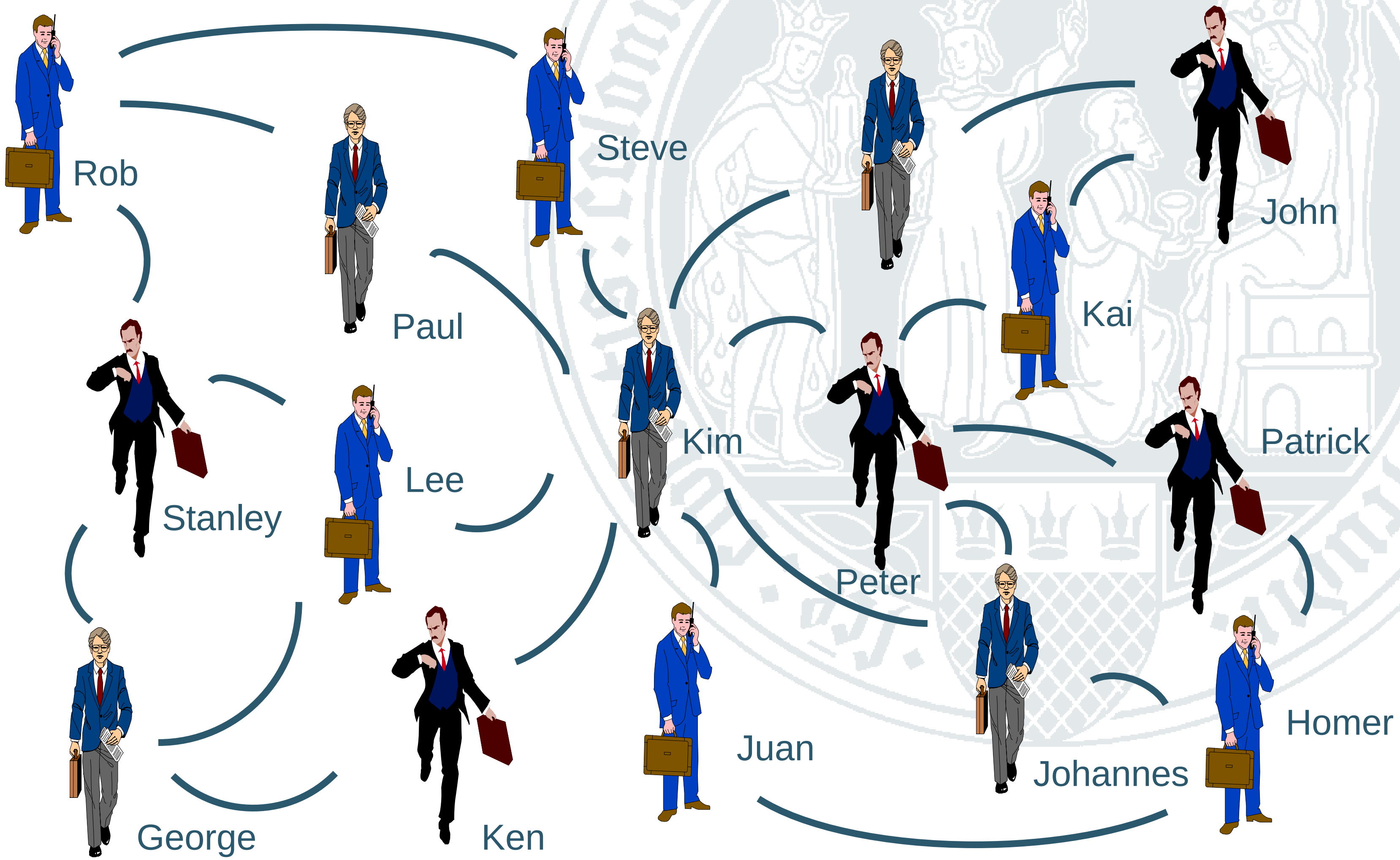


What is a Network?

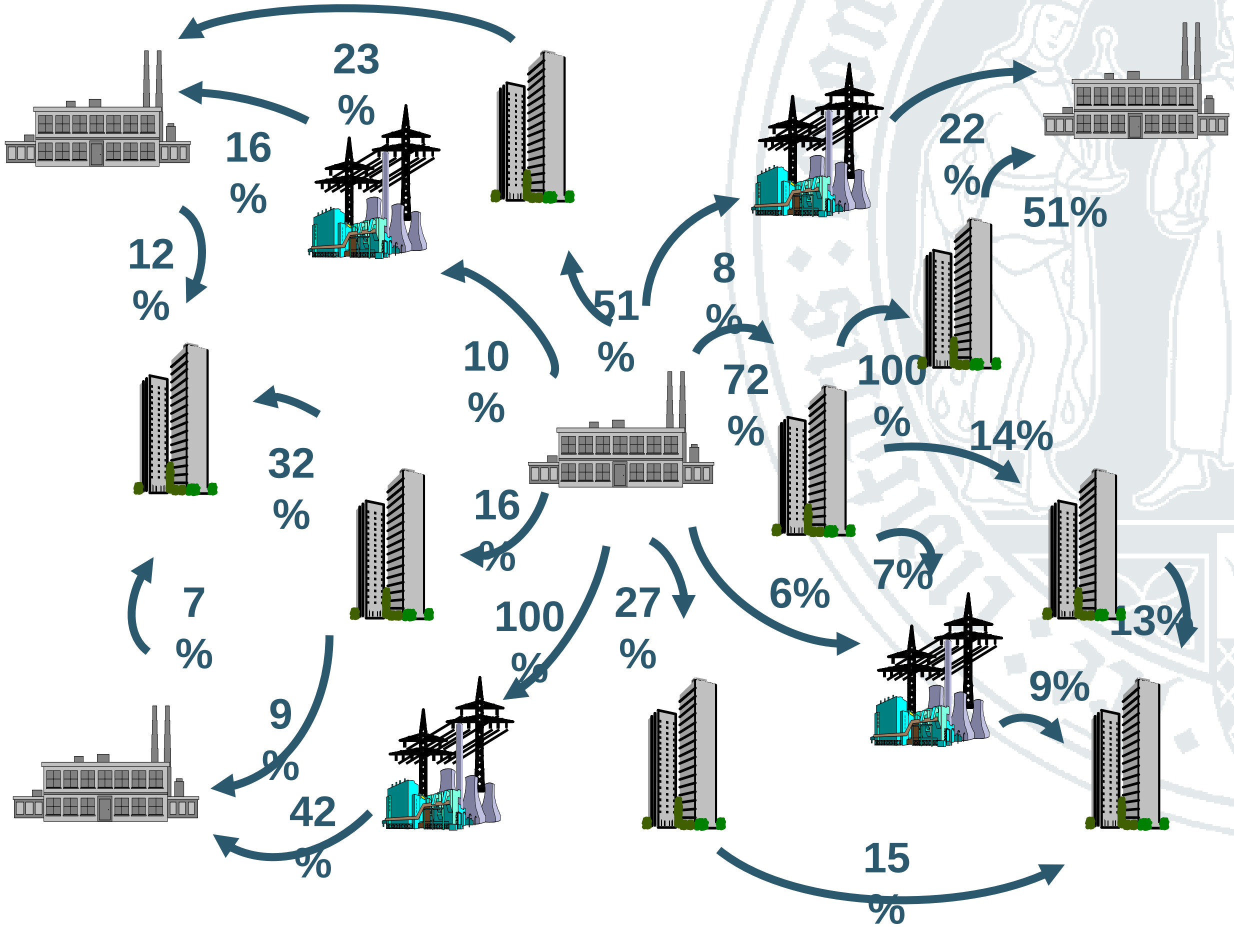
- Actors / nodes / vertices / points
- **Ties / edges / arcs / lines / links**
  - **connect pair of actors**
  - **types of social relations**
    - friendship
    - acquaintance
    - kinship
    - advice
    - hindrance
    - sex
  - **allow different kind of flows**
    - messages
    - money
    - diseases



What is a Social Network? - Relations among People



What is a Network? - Relations among Institutions

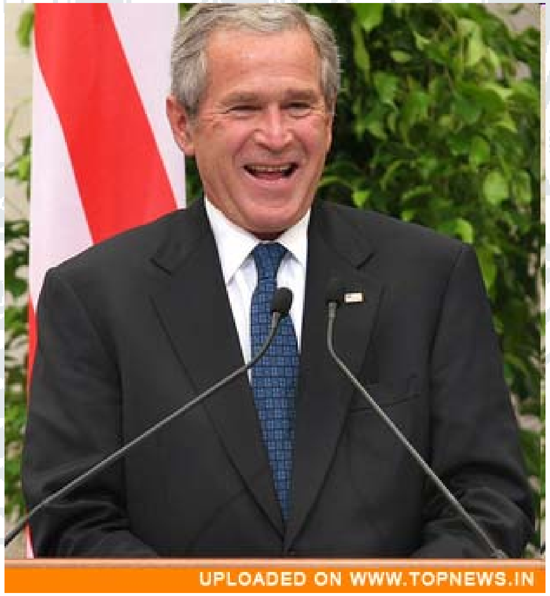


- as institutions
  - owned by, have partnership / joint venture
  - purchases from, sells to
  - competes with, supports
- through stakeholders
  - board interlocks
  - Previously worked for

Why study social networks?

Example 1) Social Capital

- **Actor's embeddedness in a social network determines opportunities and constraints an actor encounters**



- **network → social capital**

Example 2) Homophily Theory

	Male	Female
Male	123	68
Female	95	164

- Birds of a feather flock together
- See McPherson, Smith-Lovin & Cook (2001)

	0-13	14-29	30-44	45-65	>65
0-13	212	63	117	72	91
14-29	83	372	75	67	84
30-44	105	98	321	214	117
45-65	62	72	232	412	148
>65	90	77	124	153	366

- age / gender → network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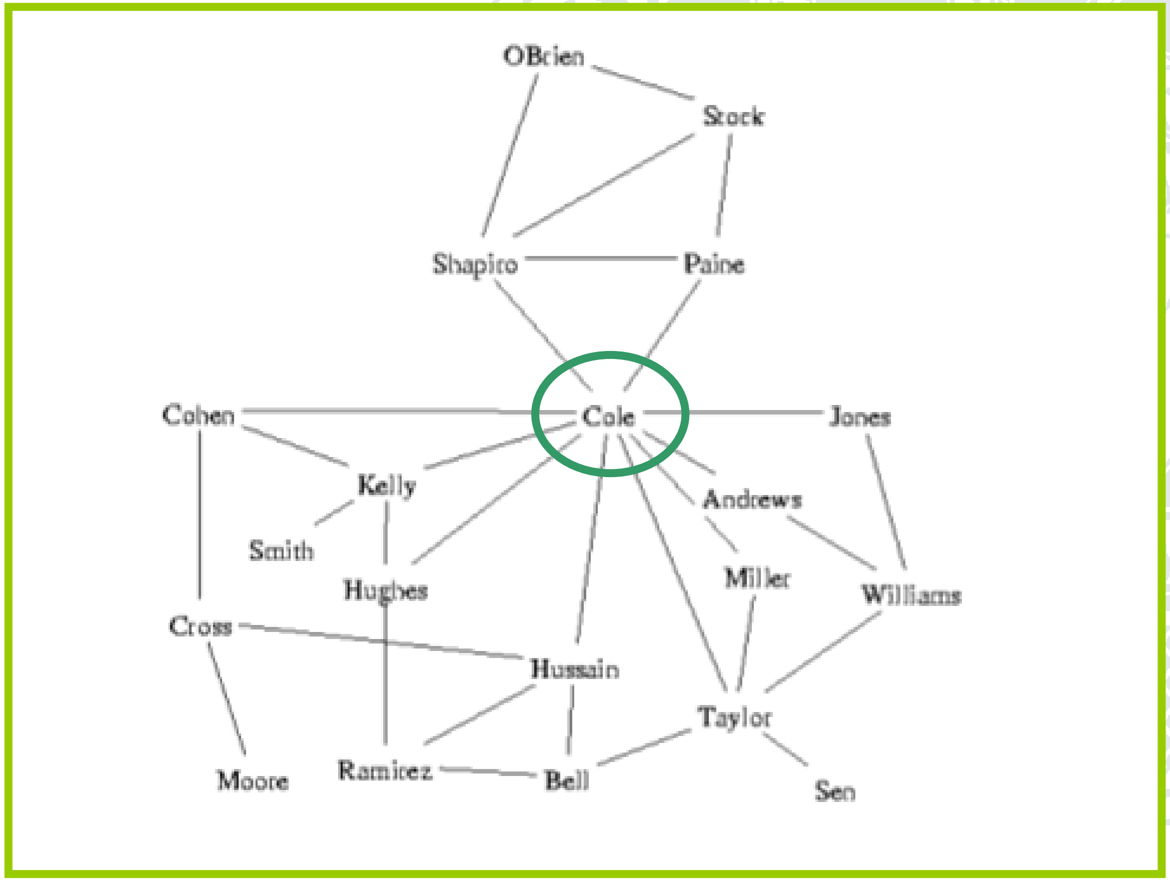
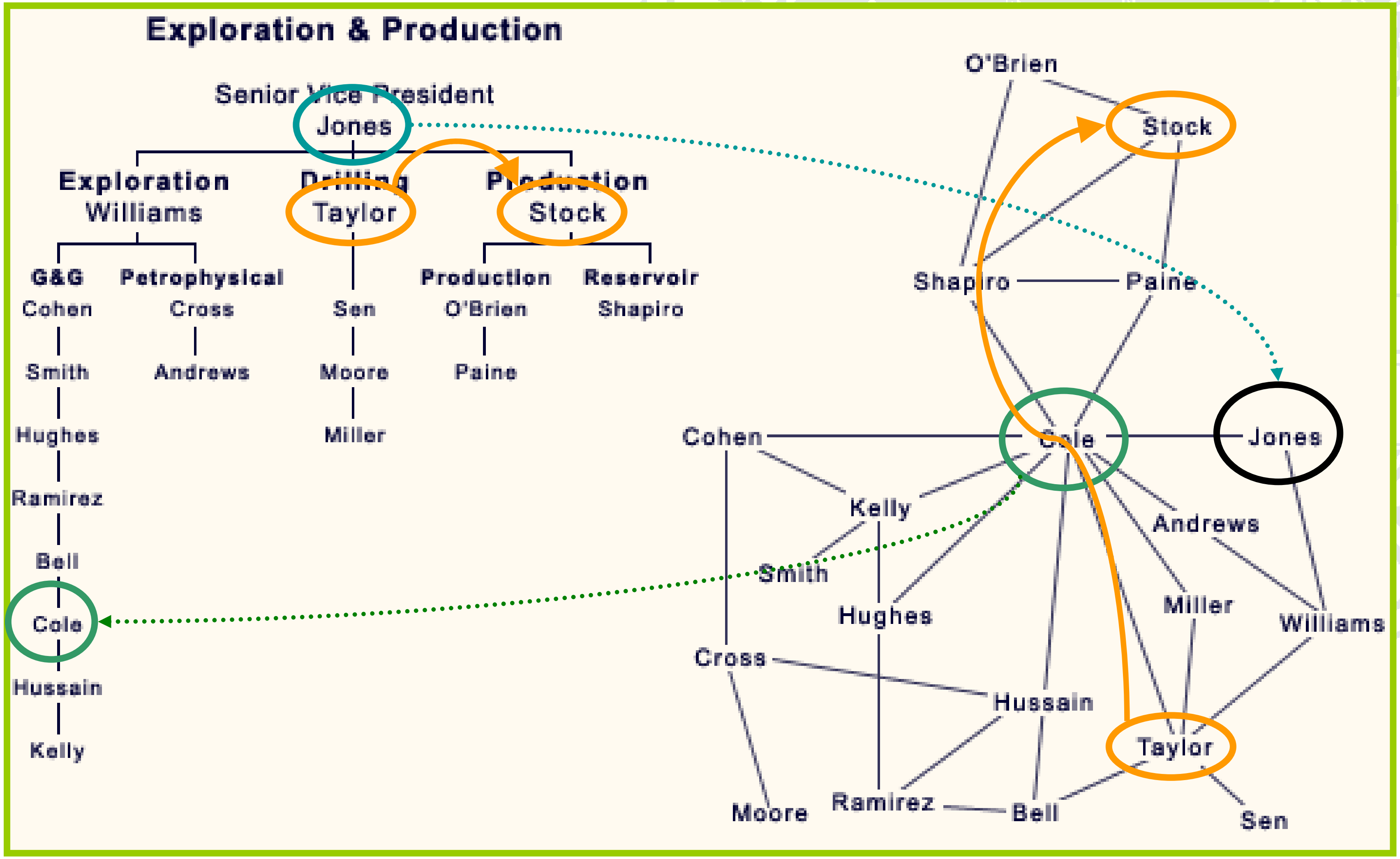
Managerial Relevance – Social Network…



Source: http://www.robcross.org/sna.htm

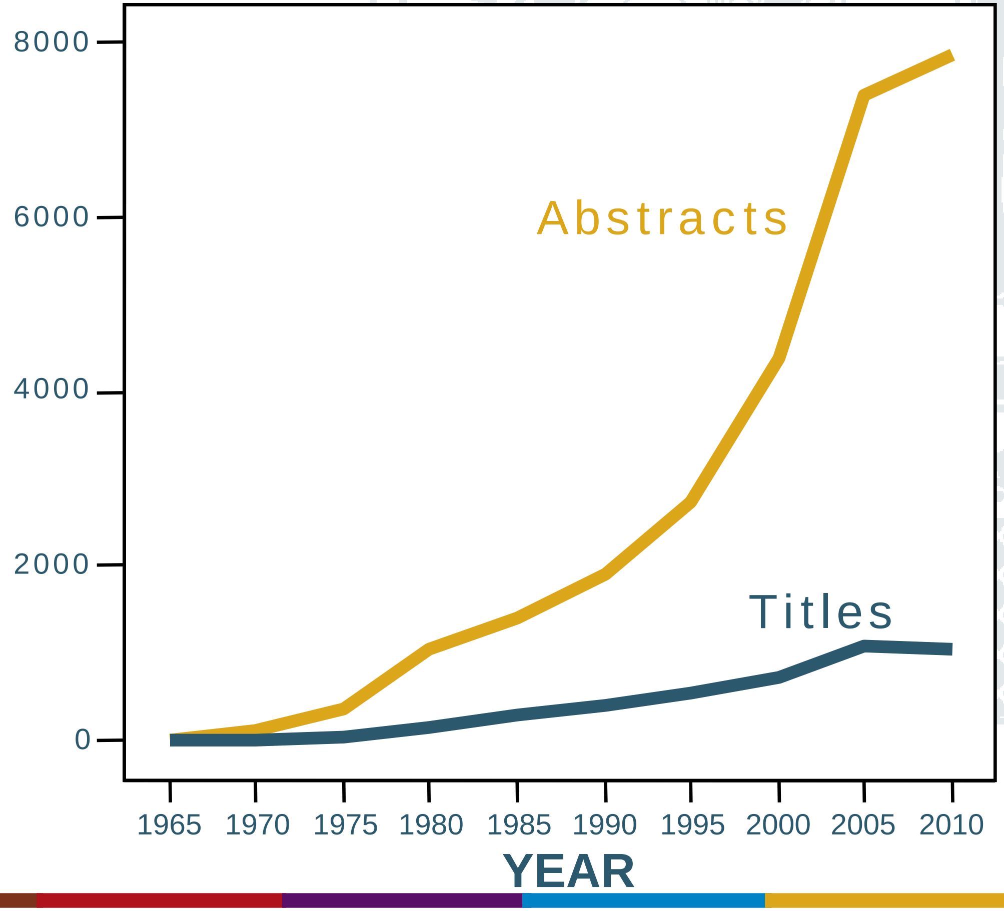
…vs. Organigram



Source: http://www.robcross.org/sna.htm

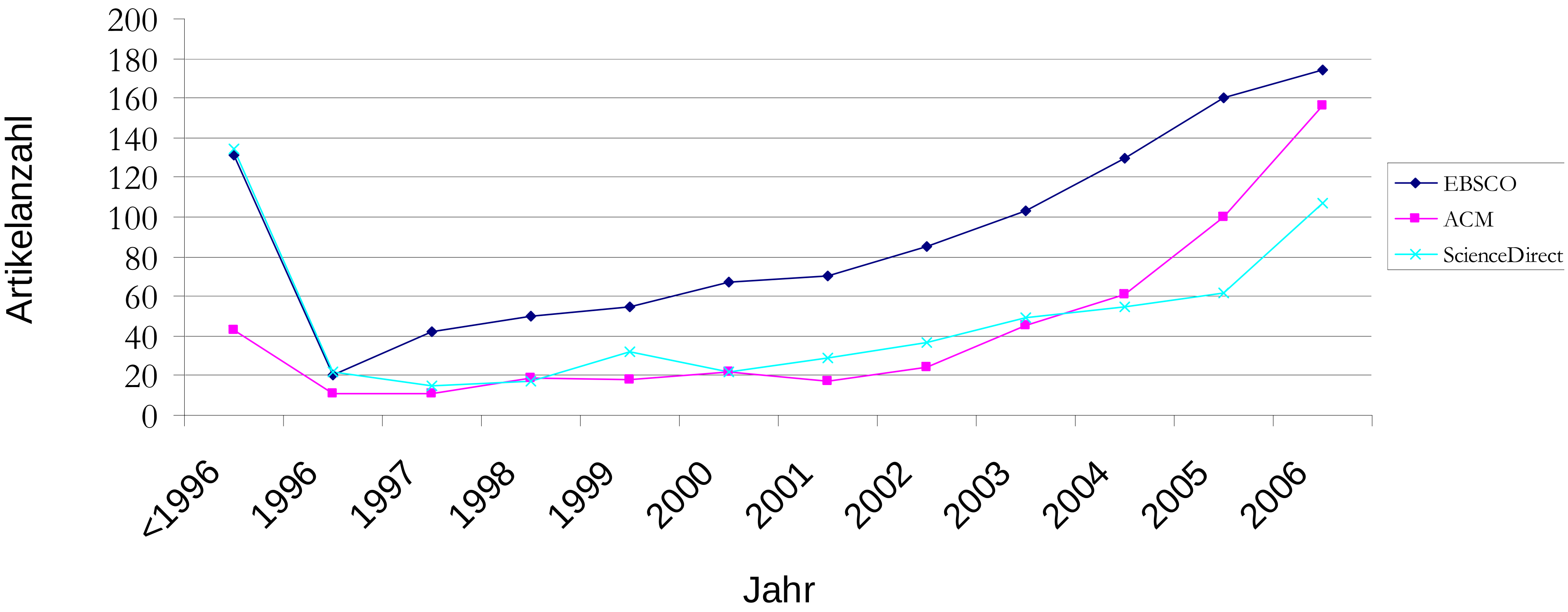
SNA – A Recent Trend in Social Sciences Research

- **Keyword search for„social“ + „network“ in 14 literature databases**



***Source:** Knoke, David (2007) Introduction to Social Network Analysis*

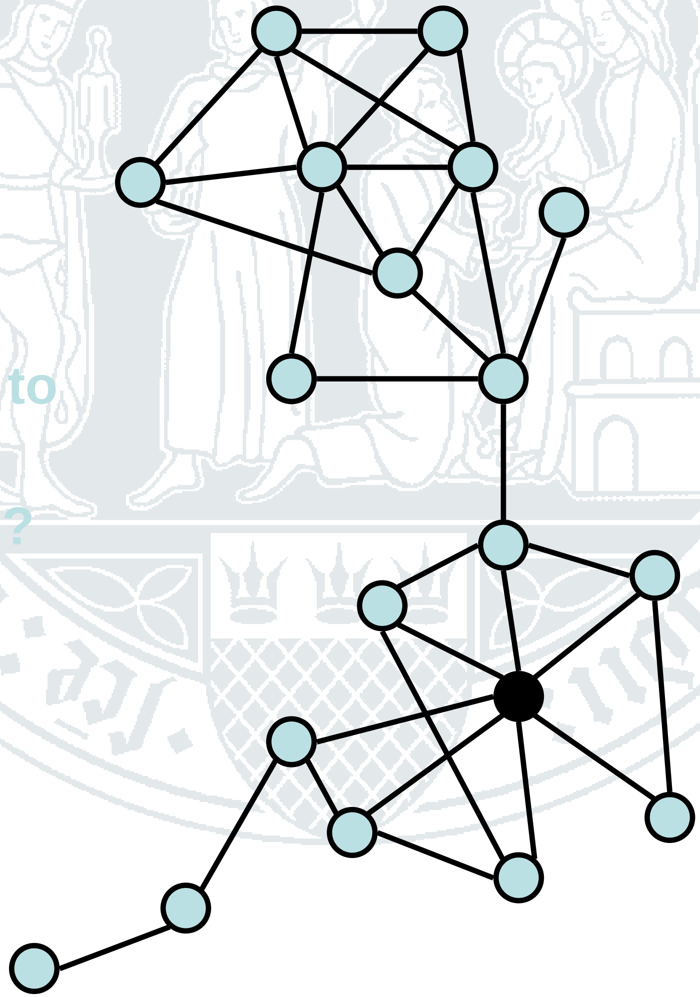
SNA – A Recent Trend in IS Research



How to analyze Social Networks?

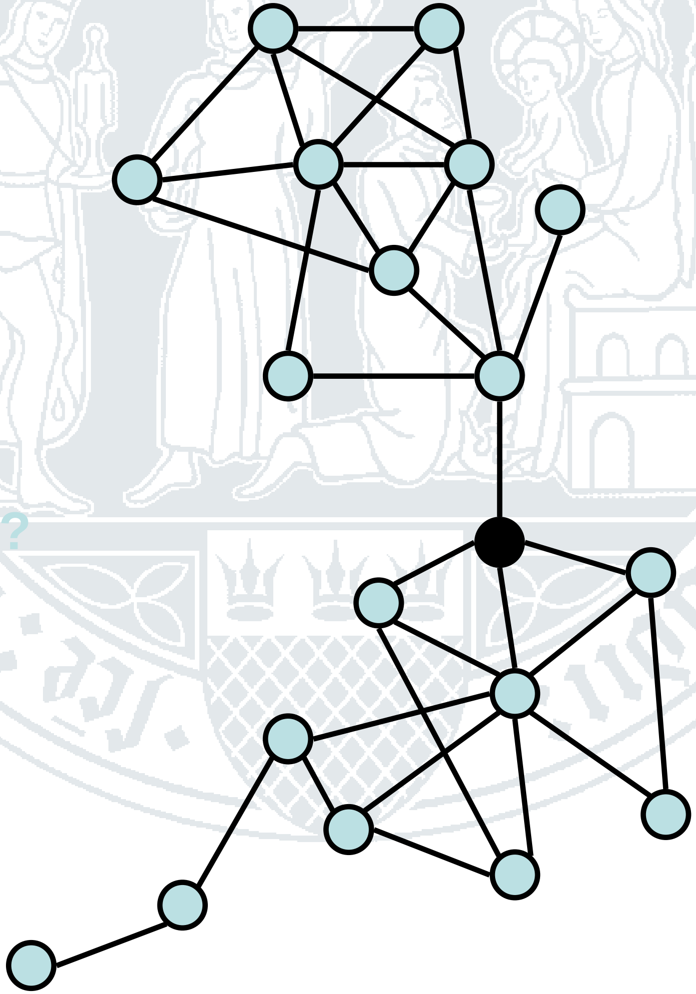
Example: Centrality Measures

- **Who is the most prominent?**
  - ***Who knows the most actors? (Degree Centrality)***
  - Who has the shortest distance to the other actors?
  - Who controls knowledge flows?
  - ...



Example: Centrality Measures

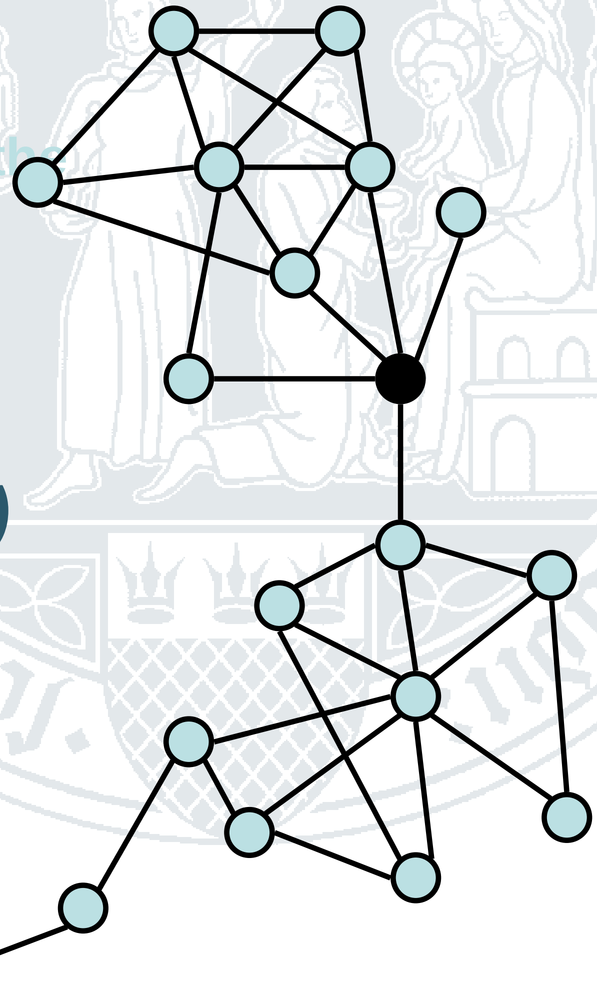
- **Who is the most prominent?**
  - Who knows the most actors?
  - ***Who has the shortest distance to the other actors? (Closesness Centrality)***
  - Who controls knowledge flows?
  - ...



Example: Centrality Measures

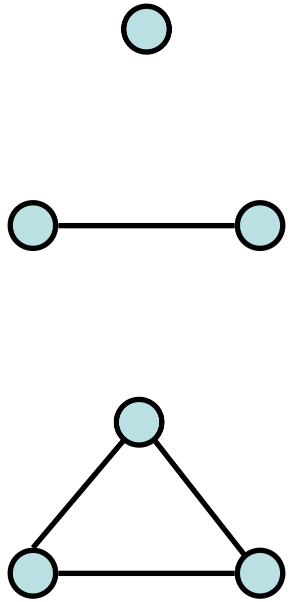
- **Who is the most prominent?**
  - Who knows the most actors
  - Who has the sthortest distance to the other actors?
  - ***Who controls knowledge flows?***

    (Betweenness Centrality)
  - ...



Basic Concepts

Dyads, Triads and Relations

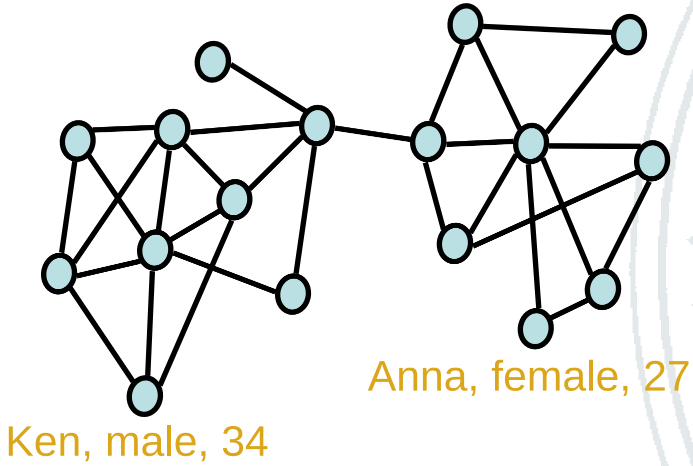


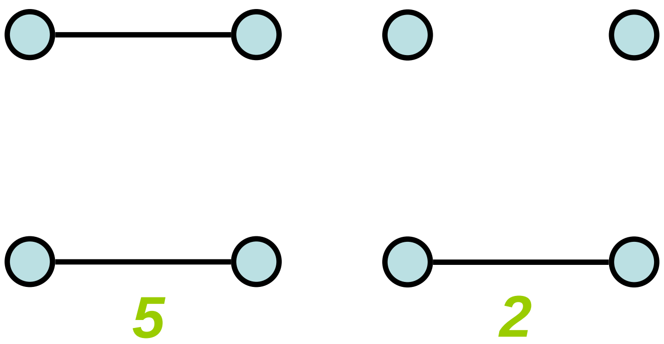
friendship

kinship

- *actor*
- *dyad*
- *triad*
- *relation:*
  - collection of specific ties among members of a group

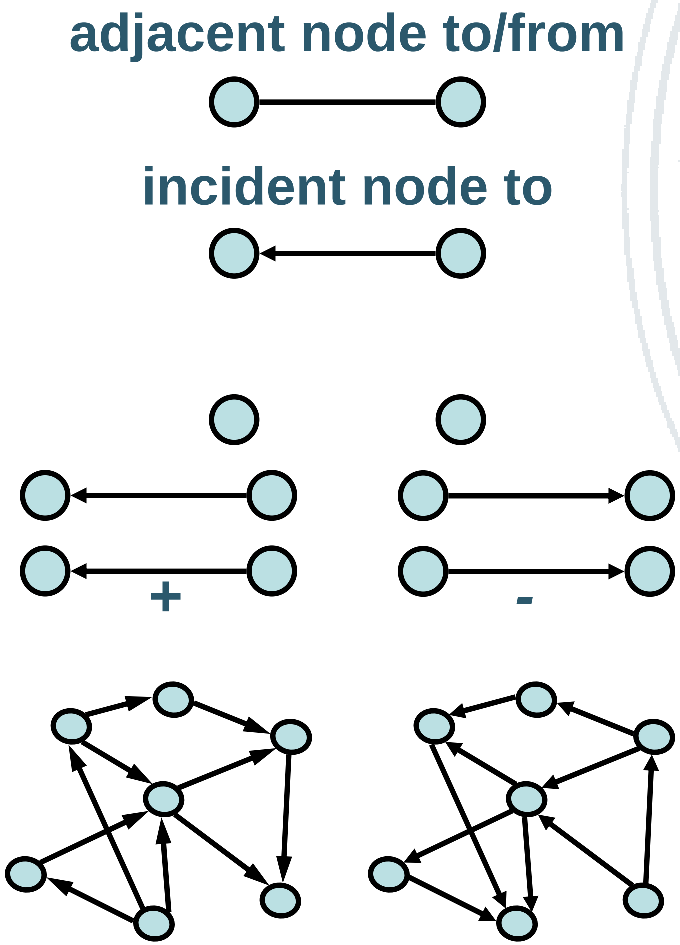
Strength of a Tie





- ***Social network***
  - finite set of actors and relation(s) defined on them
  - depicted in ***graph/ sociogram***
    - *labeled graph*
- ***Strength of a Tie***
  - ***dichotomous* vs. *valued***
    - depicted in *valued graph* or *signed graph* (+/-)

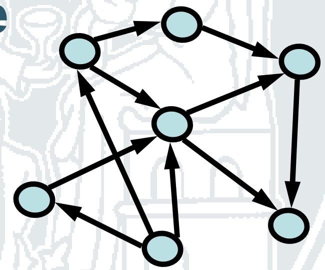
Strength of a Tie



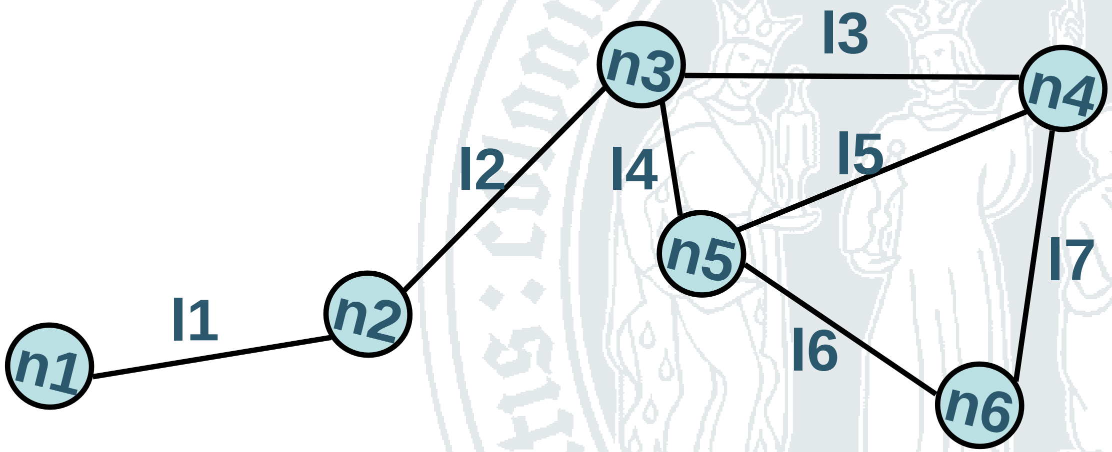
- ***Strength of a Tie***
  - ***nondirectional*** **vs.** ***directional***
    - depicted in *directed graphs (digraphs)*
    - nodes connected by *arcs*
    - 3 *isomorphism classes*
      - › ***null dyad***
      - › ***mutual / reciprocal / symmetrical dyad***
      - › ***asymmetric / antisymmetric dyad***
    - *converse of a digraph*
      - › ***reverse direction of all arcs***

Walks, Trails, Paths

- ***(Directed) Walk (W)***
  - sequence of nodes and lines starting and ending with (different) nodes (called *origin* and *terminus*)
  - Nodes and lines can be included more than once
- ***Inverse*** **of a (directed) walk** (W^{-1})
  - Walk in opposite order
- ***Length*** **of a walk**
  - How many lines occur in the walk? (same line counts double, in weighted graphs add line weights)
- ***(Directed) Trail***
  - Is a walk in which all lines are distinct
- ***(Directed) Path***
  - Walk in which all nodes and all lines are distinct
- **Every path is a trail and every trail is a walk**



Walks, Trails and Paths - Repetition

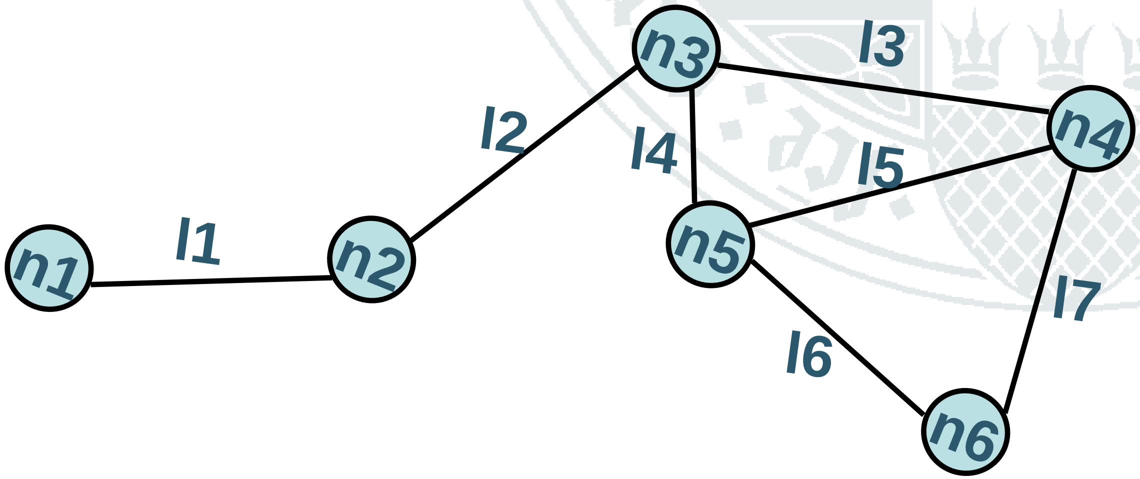


- W = n1 l1 n2 l2 n3 l4 n5 l6 n6
  - n1
  - n3
- W = n1 l1 n2 l2 n3 l4 n5 l4 n3
- W = n1 l1 n2 l2 n3 l4 n5 l5 n4 l3 n3

- Path
  - origin
  - terminus
- Walk
- Trail

Reachability, Distances and Diameter

- ***Reachability***
  - If there is a path between nodes n_i and n_j
- ***Geodesic***
  - Shortest path between two nodes
- ***(Geodesic) Distance* d(i,j)**
  - Length of Geodesic (also called „degrees of separation“)



Mathematical Notation and Fundamentals

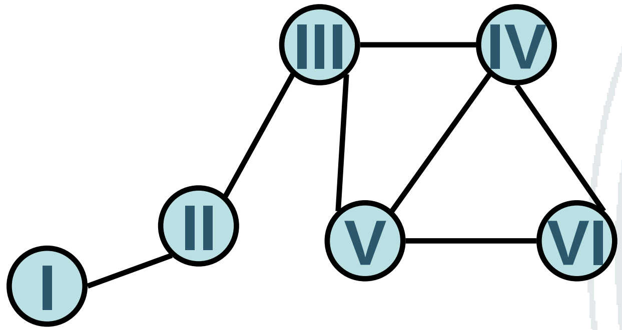
Three different notational schemes

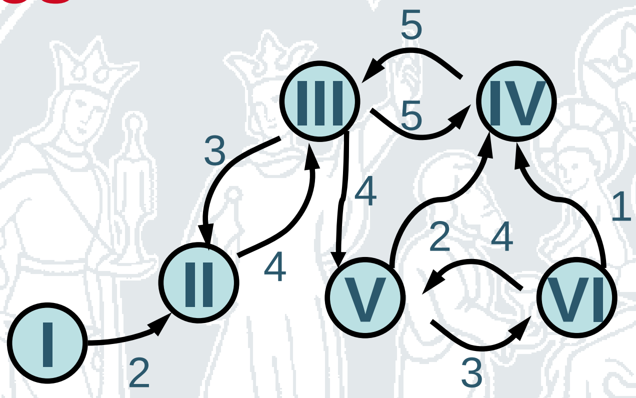
1. Graph theoretic
2. Sociometric
3. Algebraic

1. Graph Theoretic Notation

- **N Actors** — $\{n_1, n_2, \ldots, n_g\}$
- $n_1 \rightarrow n_j$ — there is a tie between the ordered pair $<n_i, n_j>$
- $n_1 \nrightarrow n_j$ — there is no tie
- (n_i, n_j) — nondirectional relation
- $<n_i, n_j>$ — directional relation
- $g(g-1)$ — number of ordered pairs in $<n_i, n_j>$ directional network
- $g(g-1)/2$ — number of ordered pairs in nondirectional network
- L — collection of ordered pairs with ties $\{l_1, l_2, \ldots, l_g\}$
- G — graph descriped by sets (N, L)
- *Simple graph* has no *reflexive ties, loops*

2. Sociometric Notation - From Graphs to (Adjacency/Socio)-Matrices





Binary, undirected

symmetrical

	I	II	III	IV	V	VI
I	-					
II	1	-				
III		1	-			
IV			1	-		
V			1	1	-	
VI				1	1	-

Valued, directed

	I	II	III	IV	V	VI
I		2	0	0	0	0
II	0	0	4	0	0	0
III	0	3	0	5	4	0
IV	0	0	5	0	0	0
V	0	0	0	2	0	3
VI	0	0	0	1	4	0

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2. Sociometric Notation

- **X** **$g \times g$ *sociomatrix* on a single relation**
  $g \times g \times R$ *super-sociomatrix* on R relations

  - X_R sociomatrix on relation R

- **$X_{ij(r)}$ value of tie from n_i to n_j (on relation χ_r) where $i \neq j$**

2. Sociometric Notation – From Matrices to Adjacency Lists and Arc Lists

	I	II	III	IV	V	VI
I	-	1				
II	1	-	1			
III		1	-	1	1	
IV			1	-	1	1
V			1	1	-	1
VI				1	1	-

Adjacency List

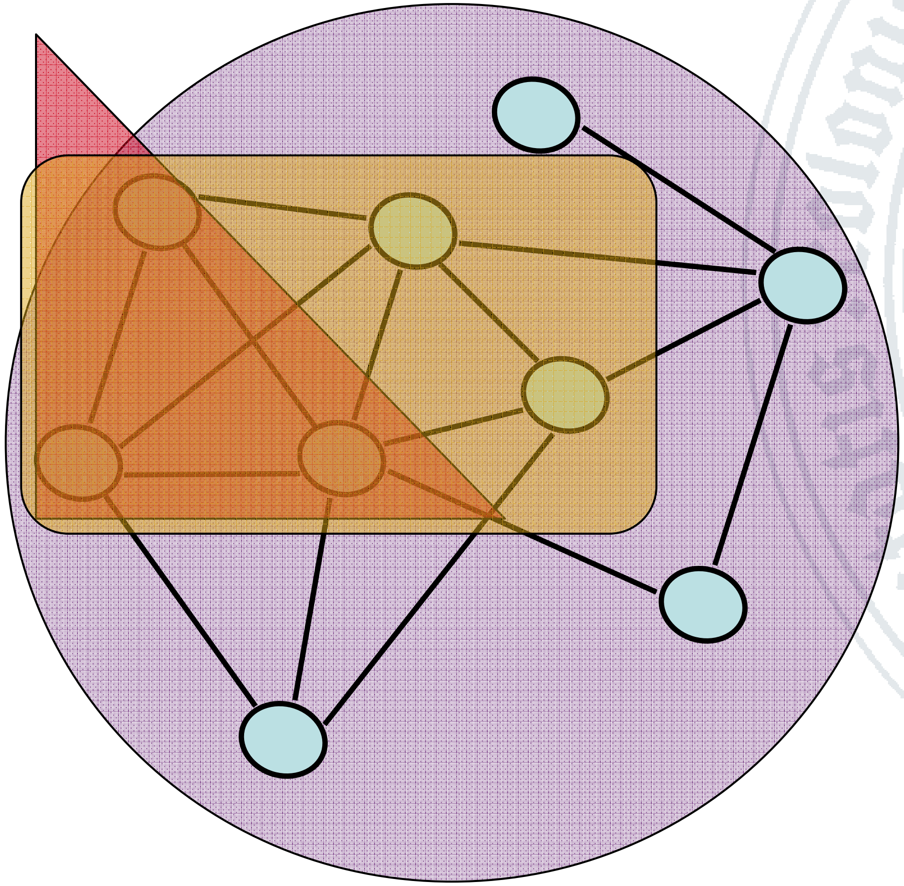
I	II
II	I III
III	II IV V
IV	III V VI
V	III V VI
VI	IV V

Arc List

I II
II I
II III
III II
III IV
III V
IV III
IV V
IV VI
V III
V IV
V VI
VI IV
VI IV

Network Statistics

Different Levels of Analysis



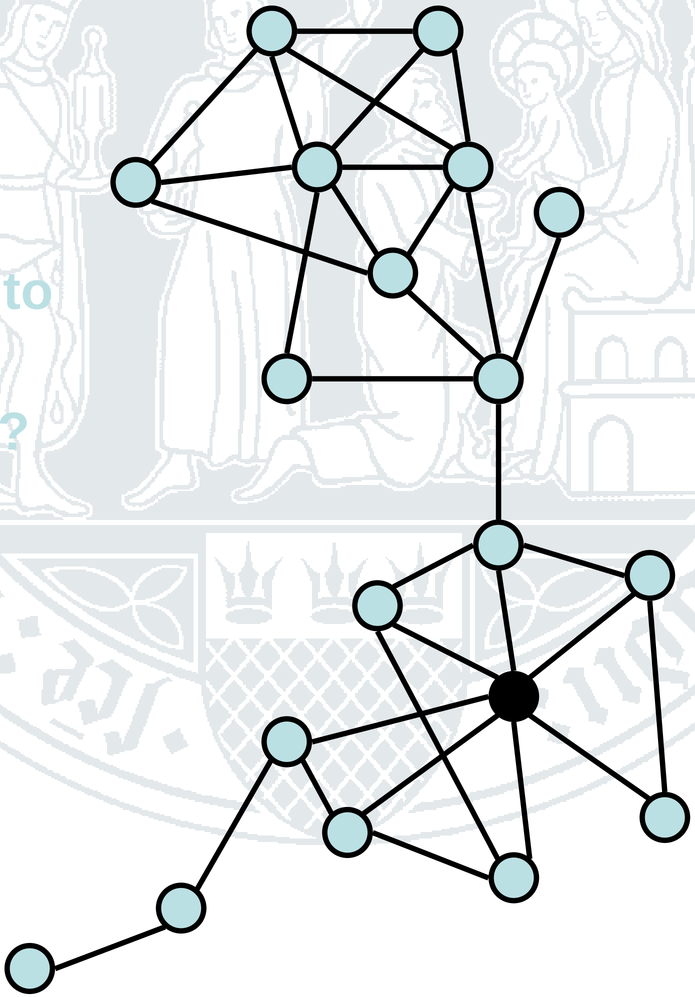
- Actor-Level
- Dyad-Level
- Triad-Level
- Subset-level (cliques / subgraphs)
- Group (i.e. global) level

Measures at the Actor-Level:
Measures of Prominence: Centrality and Prestige

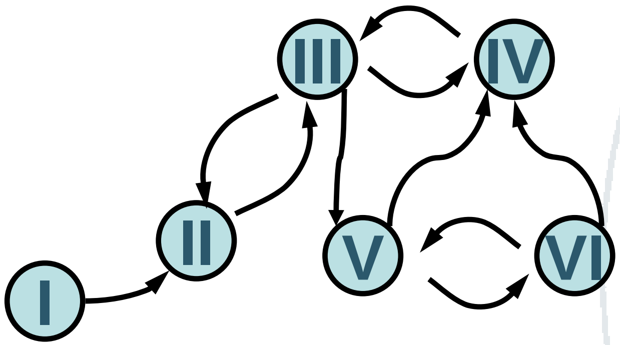
Degree Centrality

- ***Who knows the most actors?***

  (Degree Centrality)
- **Who has the shortest distance to the other actors?**
- **Who controls knowledge flows?**
- **...**



Degree Centrality I



	I	II	III	IV	V	VI	
I		1	0	0	0	0	1
II	0	0	1	0	0	0	1
III	0	1	0	1	1	0	3
IV	0	0	1	0	0	0	1
V	0	0	0	1	0	1	2
VI	0	0	0	1	0	1	2
	0	2	2	3	1	2	

- ***Indegree $d_I(n_i)$***
  - *Popularity, status, deference, degree prestige*

$$C_{DI}(n_i) = d_I(n_i) = \sum_j^g x_{ji} = x_{+i}$$

- ***Outdegree $d_O(n_i)$***
  - **Expansiveness**

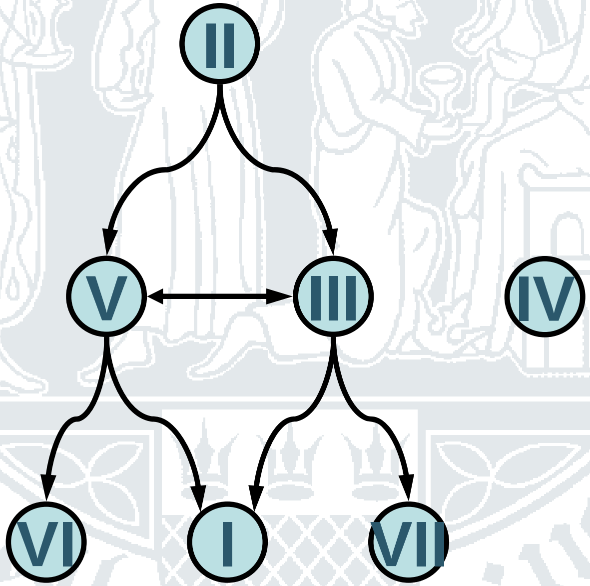
$$C_{DO}(n_i) = d_o(n_i) = \sum_j^g x_{ij} = x_{i+}$$

- **Total degree ≡ 2 x number of edges**

Marginals of adjacency matrix

Degree Centrality II

- **Interpretation: opportunity to (be) influence(d)**
- **Classification of Nodes**
  - ***Isolates***
    - $d_I(n_i) = d_O(n_i) = 0$
  - ***Transmitters***
    - $d_I(n_i) = 0$ and $d_O(n_i) > 0$
  - ***Receivers***
    - $d_I(n_i) > 0$ and $d_O(n_i) = 0$
  - ***Carriers / Ordinaries***
    - $d_I(n_i) > 0$ and $d_O(n_i) > 0$

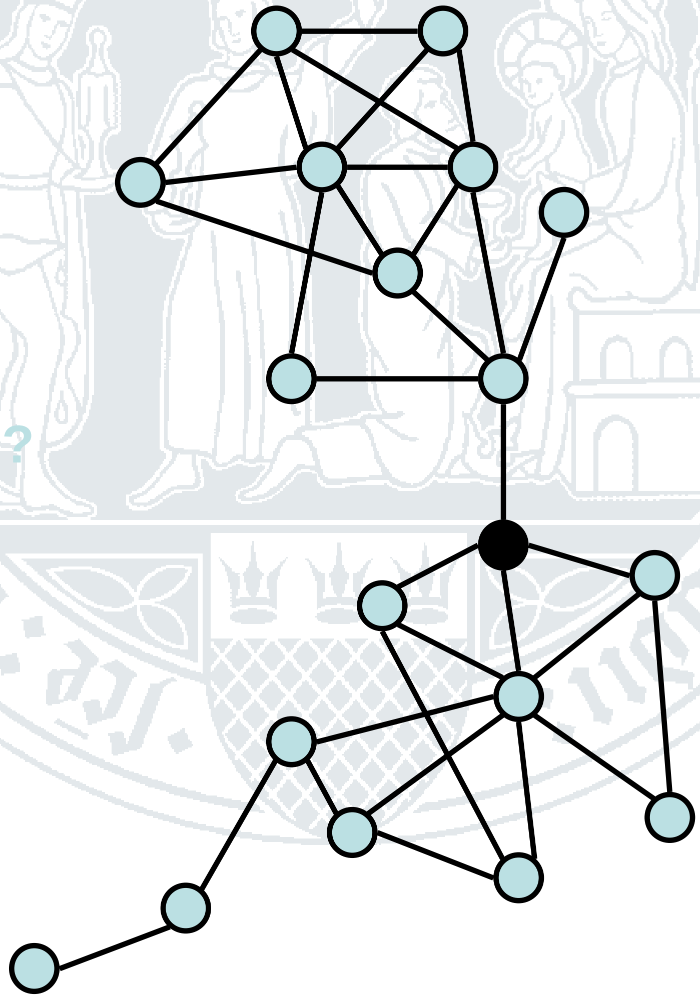


- **Standardization of C_D to allow comparison across networks of different sizes: divide by ist maximum value**

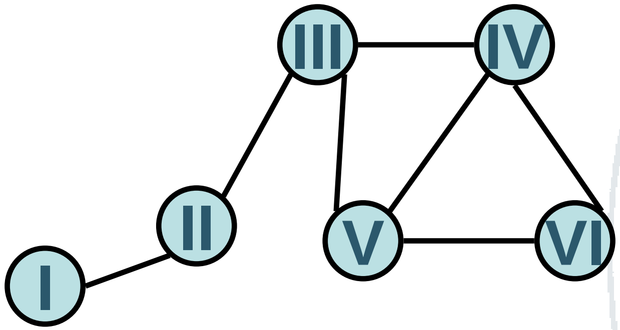
$$C_D^{'}(n_i) = \frac{d(n_i)}{g-1}$$

Closeness Centrality

- Who knows the most actors?
- ***Who has the shortest distance to the other actors? (Closesness Centrality)***
- Who controls knowledge flows?
- ...



Closeness Centrality



	I	II	III	IV	V	VI	
I	-	1	2	3	3	4	13
II	1	-	1	2	2	3	9
III	2	1	-	1	1	2	7
IV	3	2	1	-	1	1	8
V	3	2	1	1	-	1	8
VI	4	3	2	1	1	-	11

- **Index of expected arrival time**

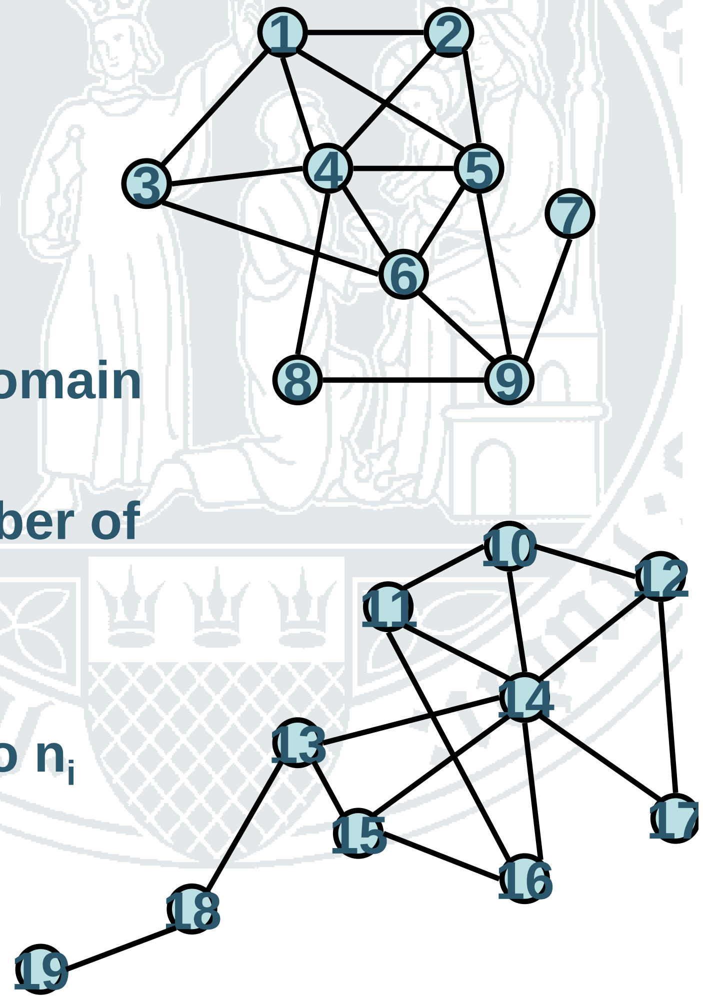
$$C_C(n_i) = \frac{1}{\sum_{j=1}^{g} d\left(n_i, n_j\right)}$$

Reciprocal of marginals of geodesic distance matrix

- **Standardize by multiplying (g-1)**
- **Problem: Only defined for connected graphs**

Proximity Prestige

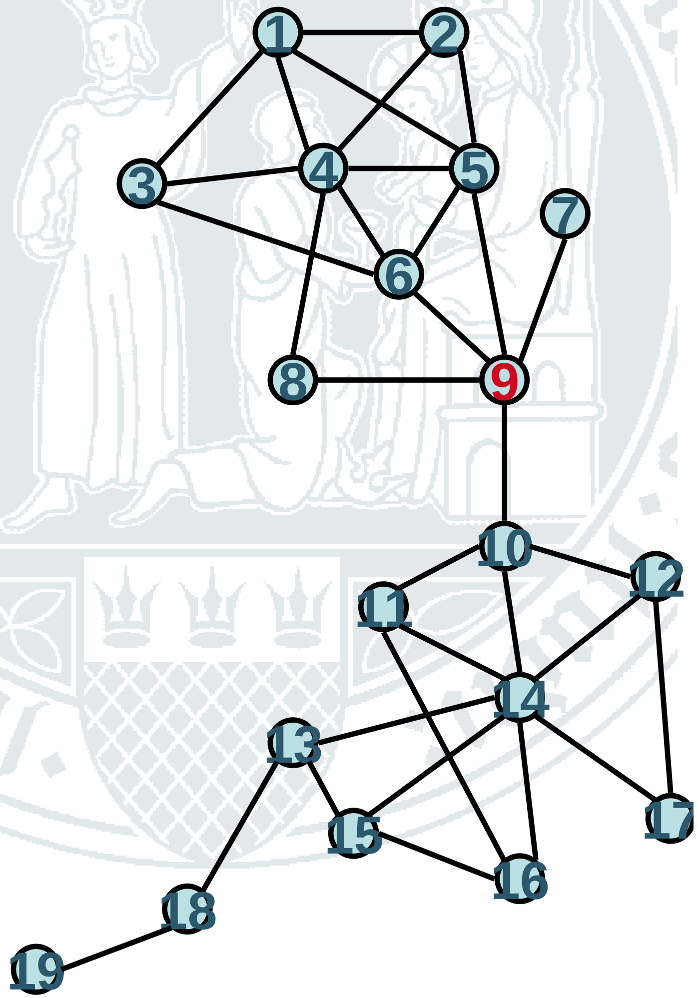
$$P_P(n_i) = \frac{I_i / (g-1)}{\sum_{j=1}^{g} d(n_j, n_i) / I_i}$$



- $I_i/(g-1)$
  - number of actors in the influence domain of n_i
  - normed by maximum possible number of actors in influence domain
- $\Sigma d(n_j,n_i)/ I_i$
  - average distance these actors are to n_i

Eccentricity / Association Number

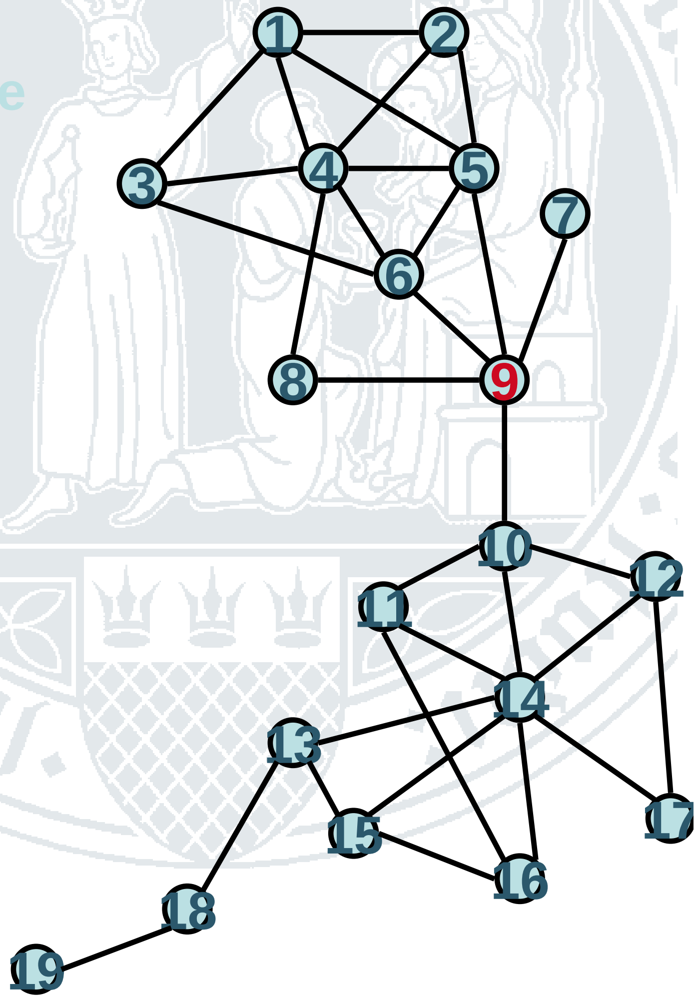
- **Largest geodesic distance between a node and any other node**
- **$\max_j d(i,j)$**



Betweenness Centrality

- Who knows the most actors?
- Who has the shortest distance to the other actors?
- ***Who controls knowledge flows?***

  (Betweenness Centrality)
- ...

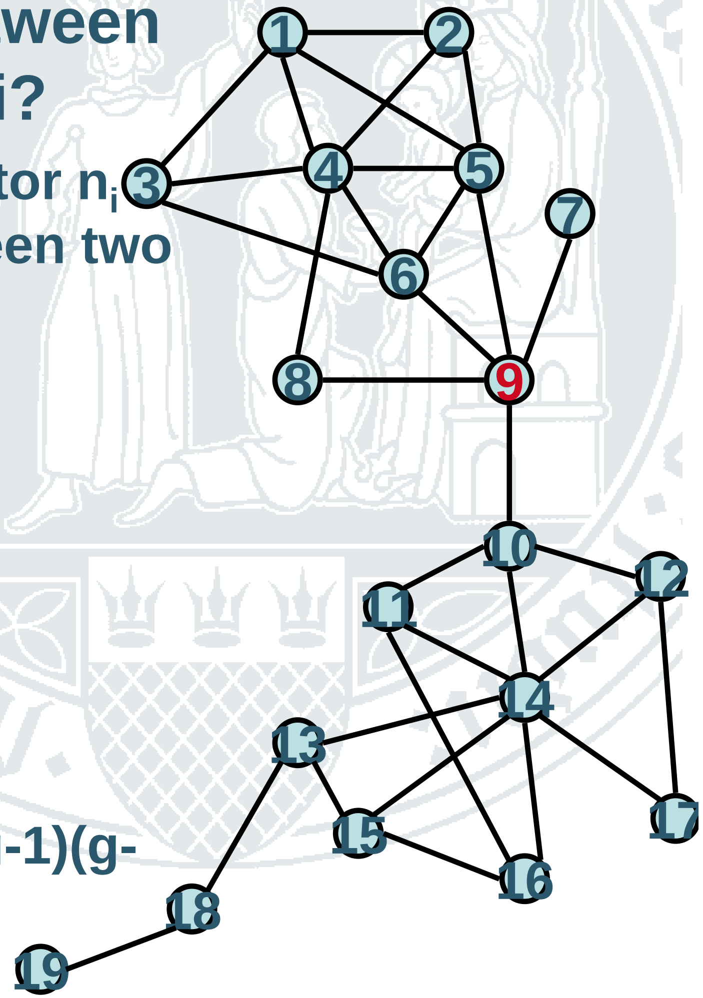


Betweenness Centrality

- **How many geodesic linkings between two actors j and k contain actor i?**
  - $g_{jk}(n_i)/g_{jk}$ probability that distinct actor n_i „involved“ in communication between two actors n_j and n_k

$$C_B(n_i) = \frac{\sum_{j<k} g_{jk}(n_i)}{g_{jk}}$$

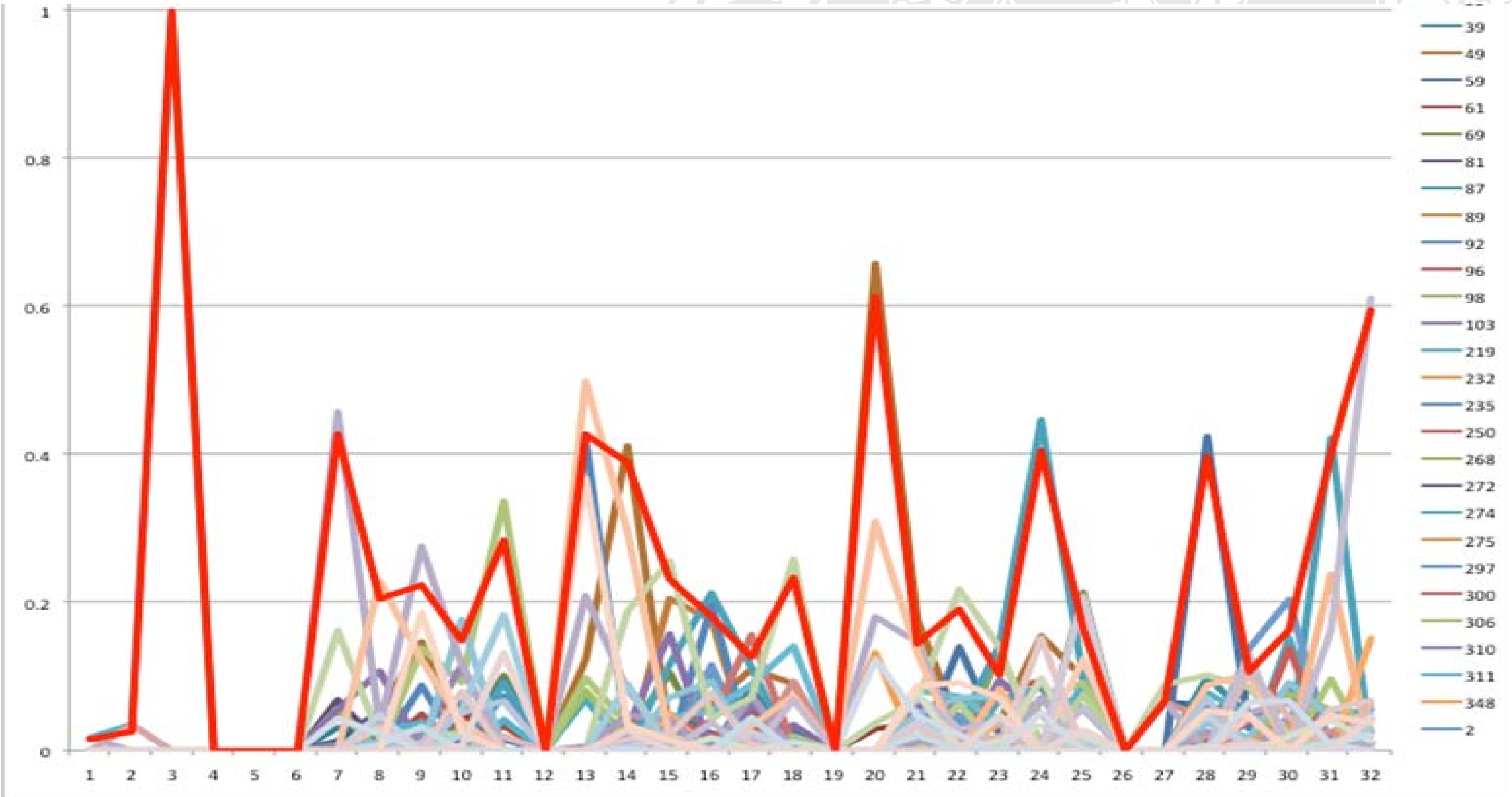
  - standardized by dividing through (g-1)(g-2)/2



Several other Centrality Measures

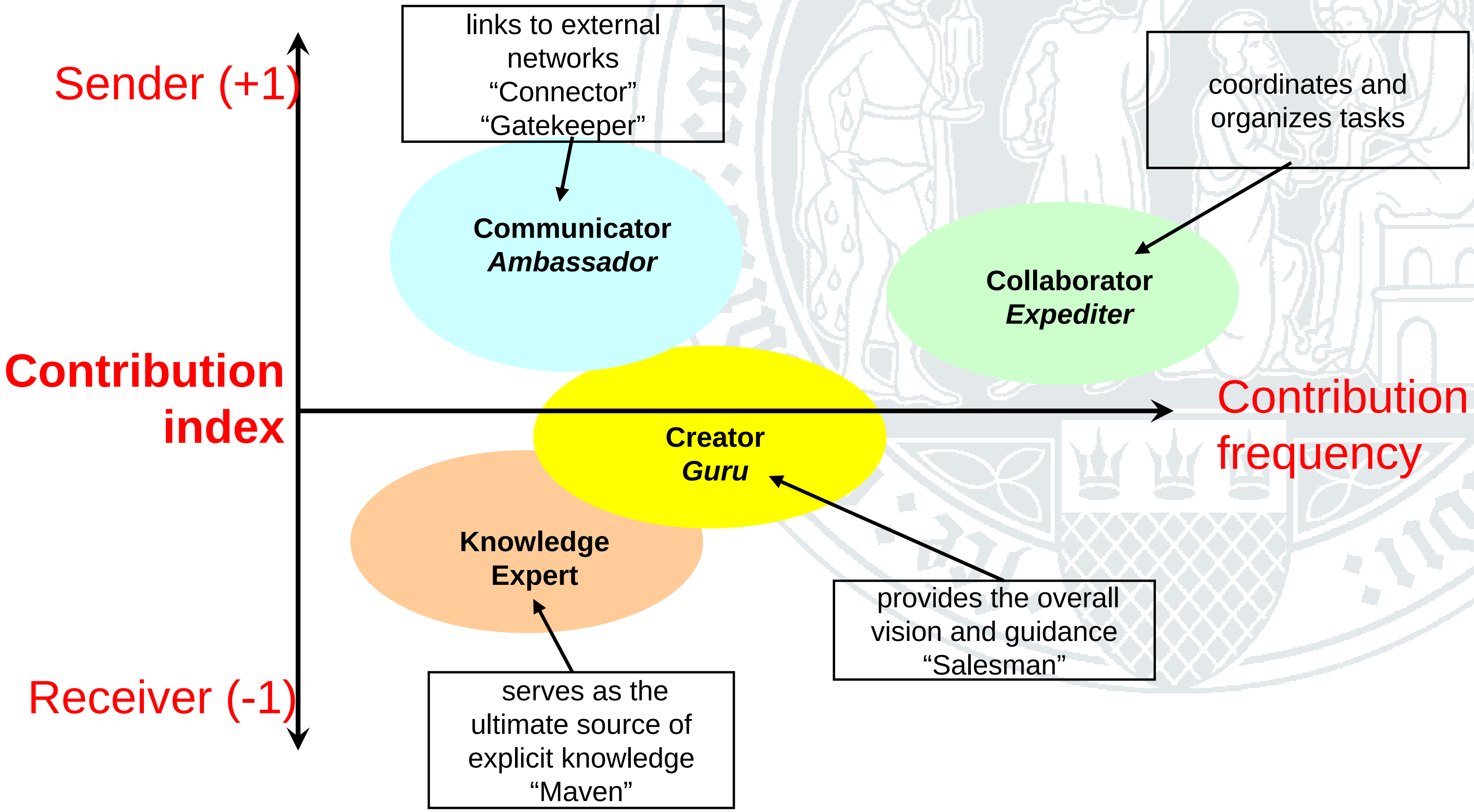
- **…beyond the scope of this lecture**
  - ***Status or Rank Prestige, Eigenvector Centrality***
    - also reflects status or prestige of people whom actor is linked to
    - Appropriate to identify *hubs* (actors adjacent to many peripheral nodes) and *bridges* (actors adjacent to few central actors)
      - **attention: more common, different meaning of bridge!!!**
  - ***Information Centrality***
  - **see Wasserman & Faust (1994), p. 192 ff.**
  - ***Random Walk Centrality***
    - see Newman (2005)

Condor – Betweenness Centrality



(Actor) Contribution Index

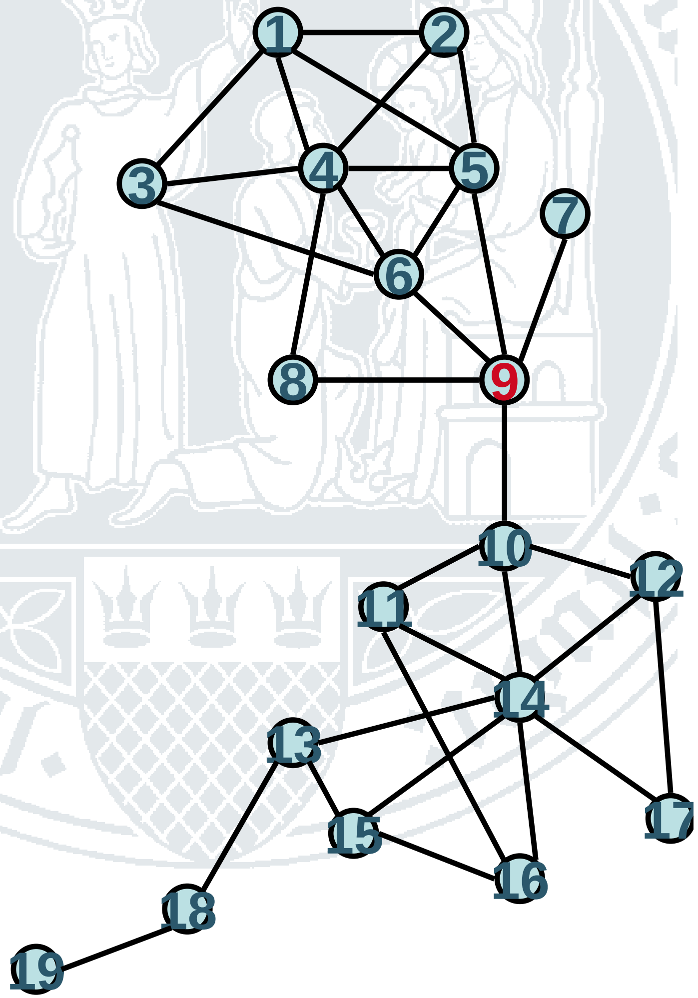
$$\frac{messages\_sent - messages\_received}{messages\_sent + messages\_received}$$



Measures at the Group-(Global-)Level and Subgroup-Level

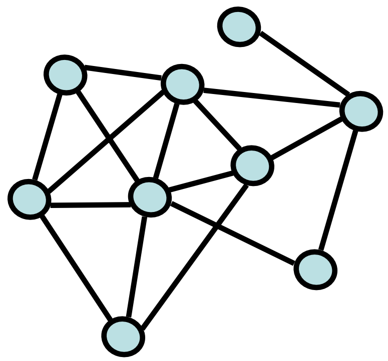
Diameter of a Graph and Average Geodesic Distance

- **Diameter**
  - Largest geodesic distance between any pair of nodes
- **Average Geodesic Distance**
  - How fast can information get transmitted?

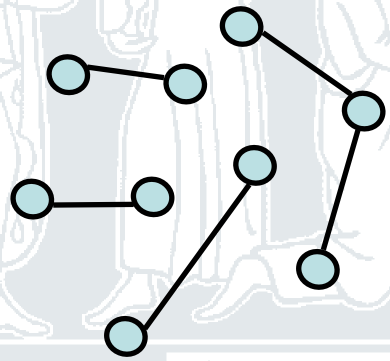


Density

- **Proportion of ties in a graph**

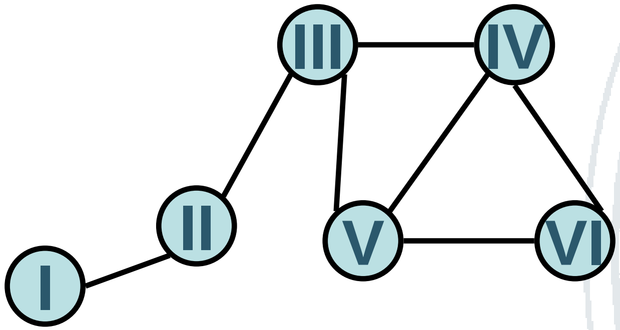


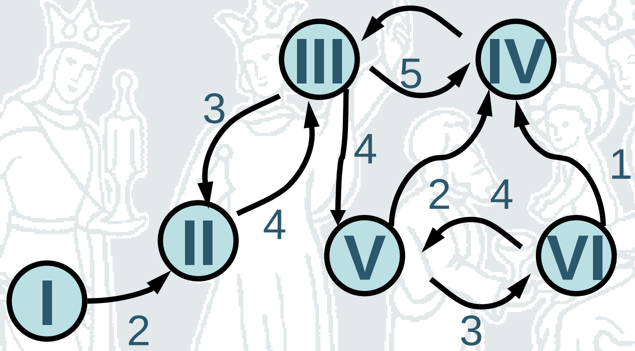
High density (44%)



Low density (14%)

Density





$$\Delta = \frac{L}{g(g-1)/2} = \frac{L}{\binom{g}{2}}$$

$$\Delta = \frac{\sum_{i=1}^{g}\sum_{j=1}^{g} x_{ij}}{g(g-1)}$$

In undirected graph:

Proportion of ties

In valued directed graph:

Average strength of the arcs

Group Centralization I

- **How equal are the individual actors' centrality values?**
  - $C_A(n_i^*)$ — actor centrality index
  - $C_A(n^*)$ — $\max_i C_A(n_i^*)$
  - $\sum_{i=1}^{g}\left[C_A(n^*) - C_A(n_i)\right]$ — sum of difference between largest value and observed values
- **General centralization index:**

$$C_A = \frac{\sum_{i=1}^{g}\left[C_A(n^*) - C_A(n_i)\right]}{\max \sum_{i=1}^{g}\left[C_A(n^*) - C_A(n_i)\right]}$$

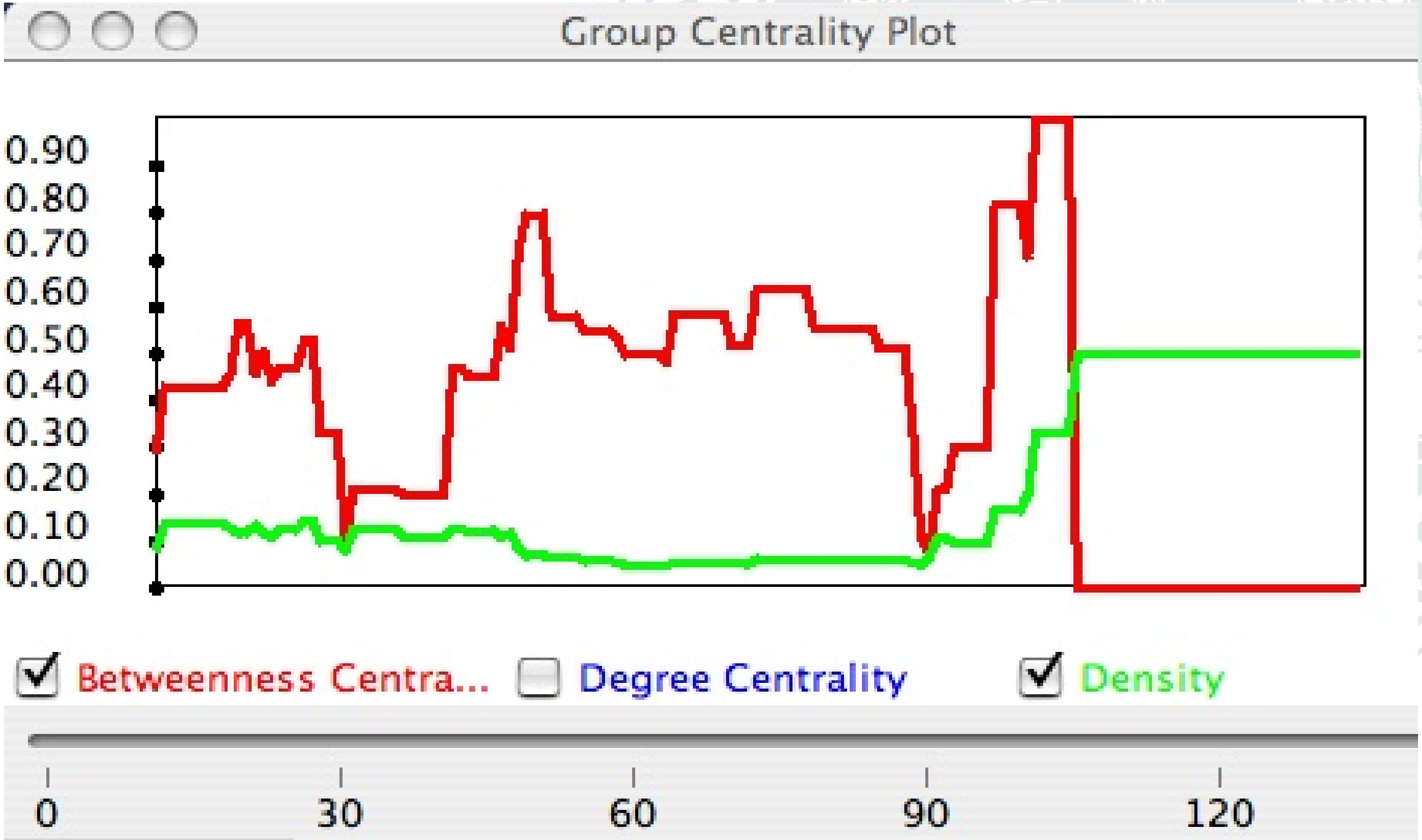
Group Centralization II

$$C_D = \frac{\sum_{i=1}^{g}\left[C_D\left(n^*\right) - C_D\left(n_i\right)\right]}{(g-1)(g-2)}$$

$$C_C = \frac{\sum_{i=1}^{g}\left[C_C^{'}\left(n^*\right) - C_C^{'}\left(n_i\right)\right]}{\left[(g-1)(g-2)\right](2g-3)}$$

$$CB = \frac{\sum_{i=1}^{g}\left[C_B\left(n^*\right) - C_B\left(n_i\right)\right]}{(g-1)^2(g-2)} = \frac{\sum_{i=1}^{g}\left[C_B^{'}\left(n^*\right) - C_B^{'}\left(n_i\right)\right]}{(g-1)}$$

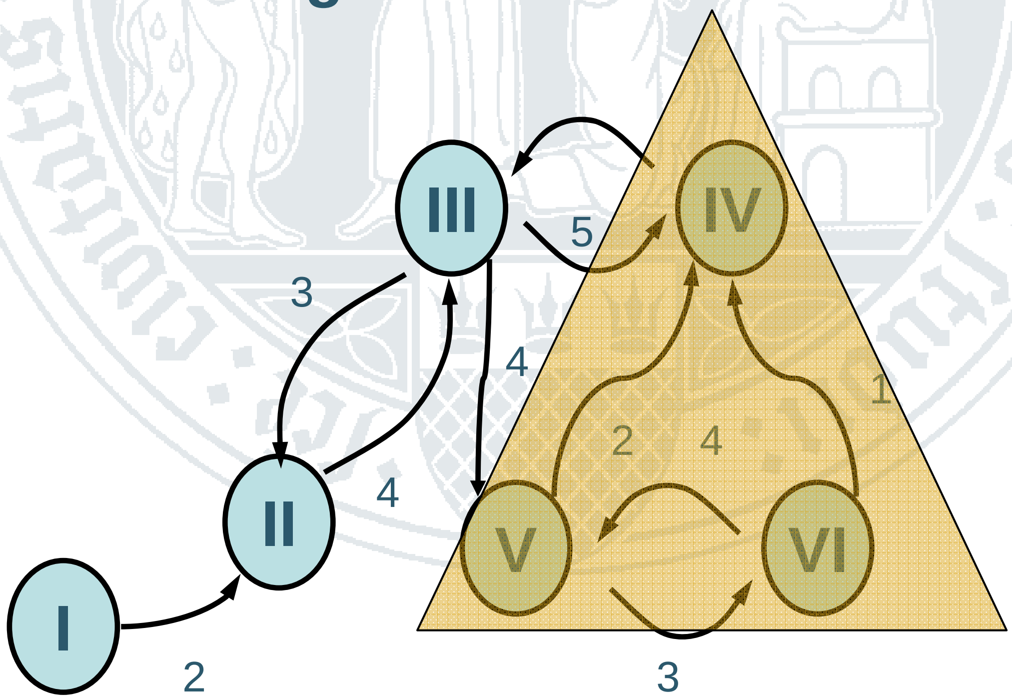
Condor – Group Centralization



Subgroup Cohesion

- average strength of ties within the subgroup divided by average strength of ties that are from subgroup members to outsiders
- >1 → ties in subgroup are stronger

$$\frac{\dfrac{\sum_{i\in N_s}\sum_{j\in N_s} x_{ij}}{g_s(g_s-1)}}{\dfrac{\sum_{i\in N_s}\sum_{j\notin N_s} x_{ij}}{g_s(g-g_s)}}$$

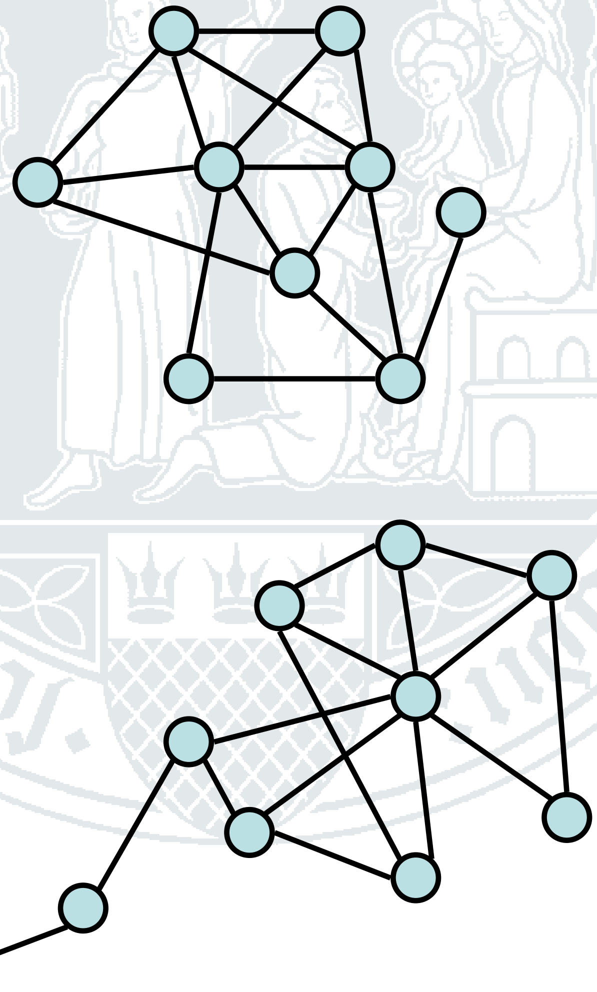


Connectivity of Graphs and Cohesive Subgroups

Connectivity of Graphs

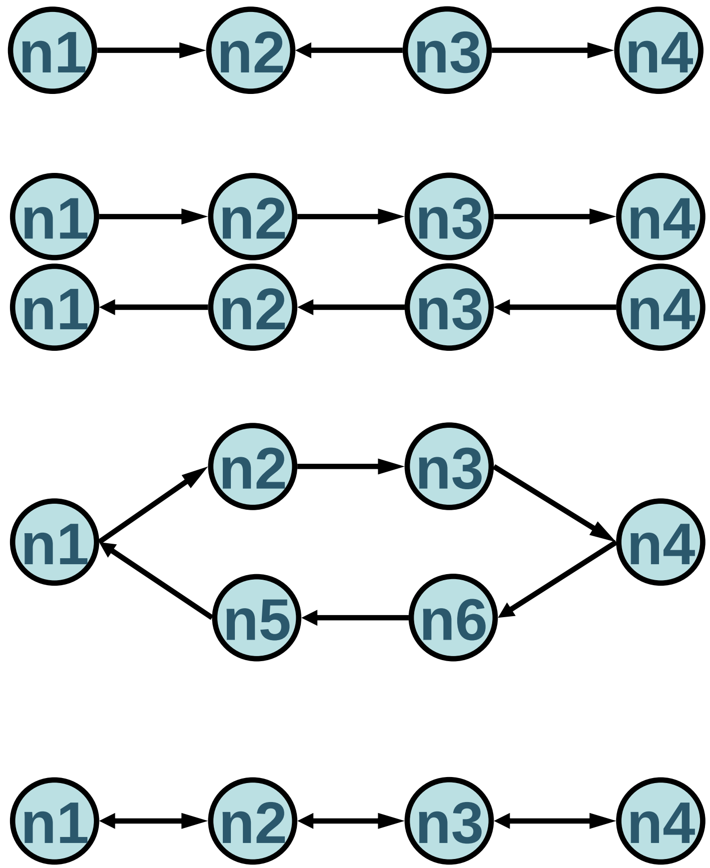
Connected Graphs, Components, Cutpoints and Bridges

- ***Connectedness***
  - A graph is connected if there is a path between every pair of nodes



- ***Components***
  - Connected subgraphs in a graph
  - Connected graph has 1 component
  - Two disconnected graphs are one social network!!!

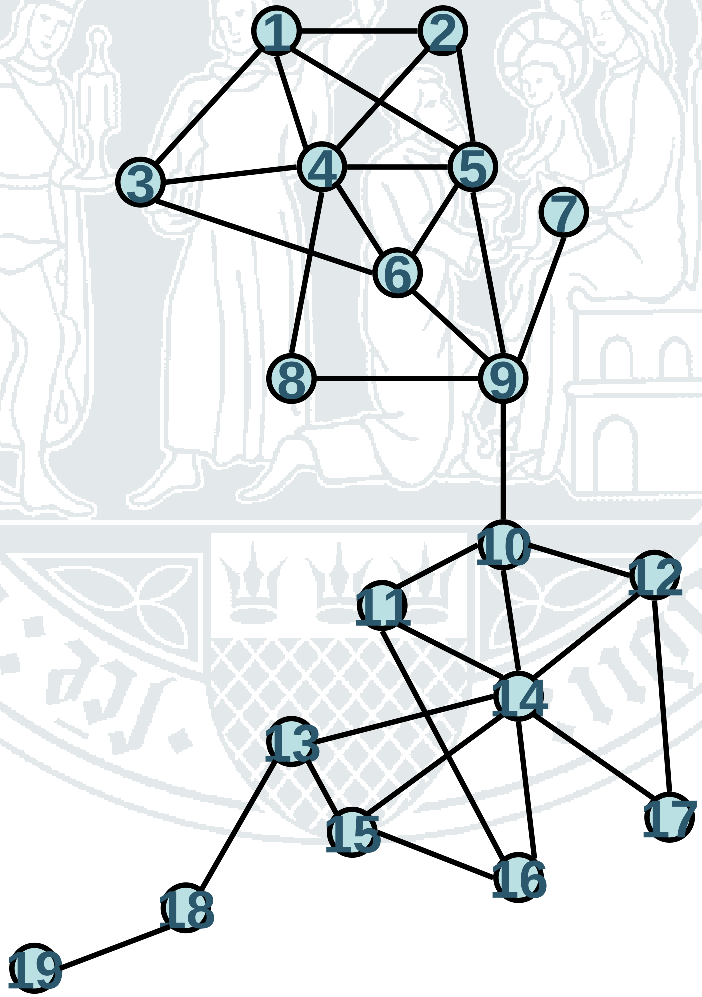
Connected Graphs, Components, Cutpoints and Bridges



- **Connectivity of pairs of nodes and graphs**
  - ***Weakly connected***
    - Joined by semipath
  - ***Unilaterally connected***
    - Path from n_j to n_j or from n_j to n_j
  - ***Strongly connected***
    - Path from n_j to n_j *and* from n_j to n_j
    - Path may contain different nodes
  - ***Recursively Connected***
    - Nodes are strongly connected an both paths use the same nodes and arcs in reverse order

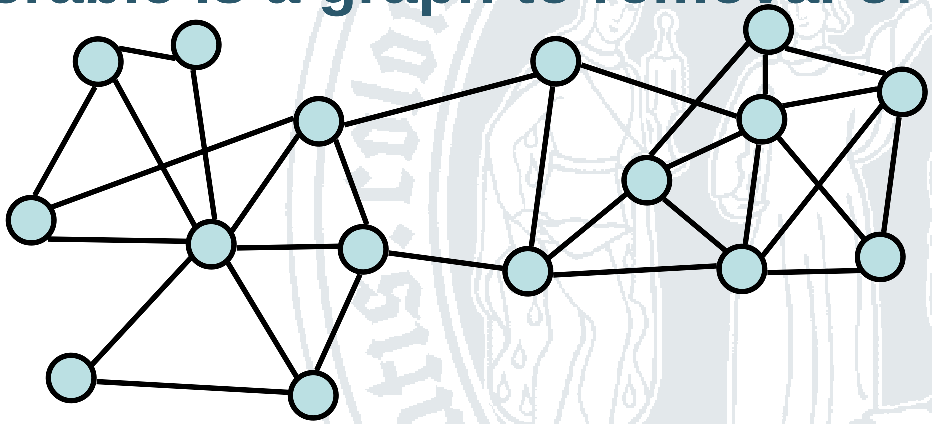
Connected Graphs, Components, Cutpoints and Bridges

- ***Cutpoints***
  - number of components in the graph that contain node n_j is fewer than number of components in subgraphs that results from deleting n_j from the graph
- ***Cutsets (of size k)***
  - *k*-node cut
- ***Bridges / line cuts***
  - Number of components…that contain line l_k



Node- and Line Connectivity

- How vulnerable is a graph to removal of nodes or lines?



Point connectivity / Node connectivity

- Minimum number of *k* for which the graph has a *k*-node cut
- For any value <k the graph is *k-node-connected*

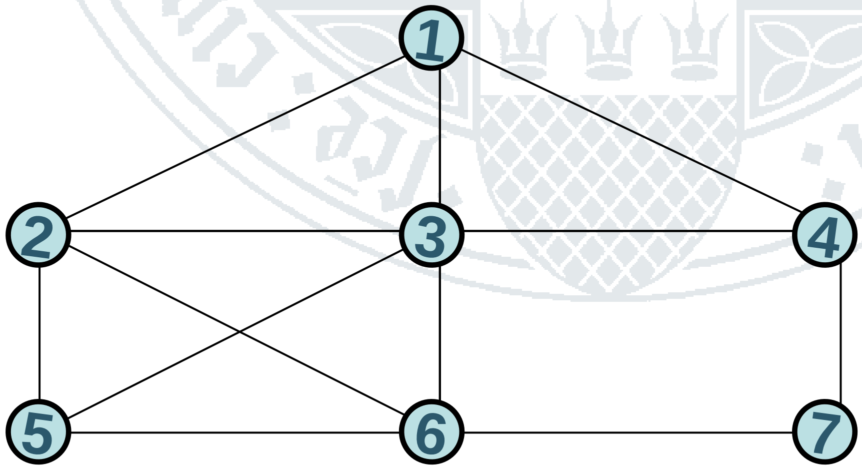
Line connectivity / Edge connectivity

- Minimum number λ for which for which graph has a λ-line cut

Cohesive Subgroups

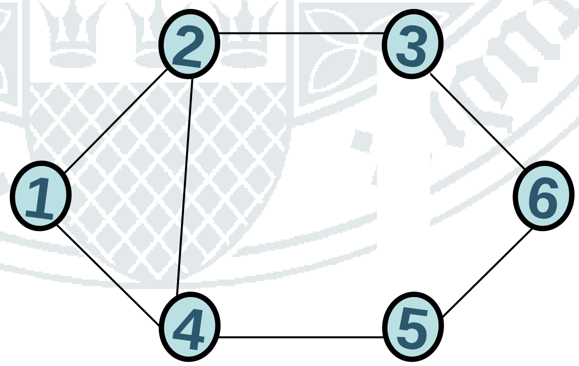
Cohesive Subgroups, (n-)Cliques, n-Clans, n-Clubs, k-Plexes, k-Cores

- ***Cohesive Subgroup***
  - Subset of actors among there are relatively strong, direct, intense, frequent or positive ties
- ***Complete Graph***
  - All nodes are adjacent
- ***Clique***
  - Maximal complete subgraph of three or more nodes
  - Cliques can overlap
  - {1, 2, 3}
  - {1, 3, 4}
  - {2, 3, 5, 6}



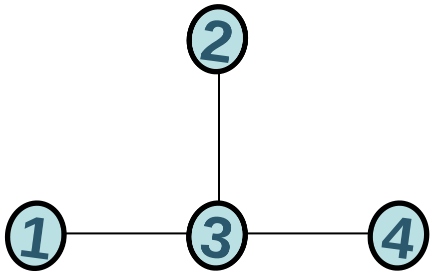
Cohesive Subgroups, (n-)Cliques, n-Clans, n-Clubs, k-Plexes, k-Cores

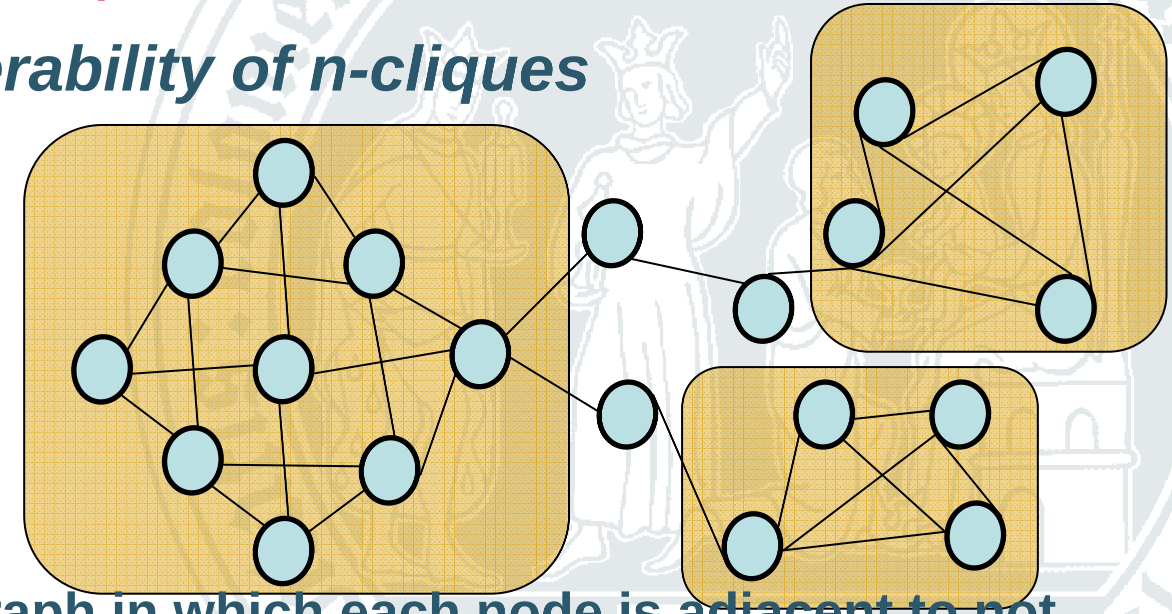
- ***n-clique***
  - maximal subgraph in which d(i,j) $\leq$ n for all n_i, n_j
  - 2: cliques: {2, 3, 4, 5, 6} and {1, 2, 3, 4, 5}
  - intermediaries in geodesics do not have to be n-clique members themselves!
- ***n-clan***
  - *n-clique* in which the d(i,j) $\leq$ n for the subgraph of all nodes in the n-clique
  - 2-clan: {2, 3, 4, 5, 6}
- ***n-club***
  - maximal subgraph of diameter n
  - 2-clubs: {1, 2, 3, 4} ; {1, 2, 3, 5} and {2, 3, 4, 5, 6}



Cohesive Subgroups, (n-)Cliques, n-Clans, n-Clubs, k-Plexes, k-Cores

- ***Problem: vulnerability of n-cliques***





- ***k-plexes***
  - **maximal subgraph in which each node is adjacent to not fewer than g_s-k nodes („maximal“: no other nodes in subgraph that also have $d_s(i) \geq (g_s - k)$]**
- ***k-cores***
  - **subgraph in which each node is adjacent to at least k other nodes in the subgraph**

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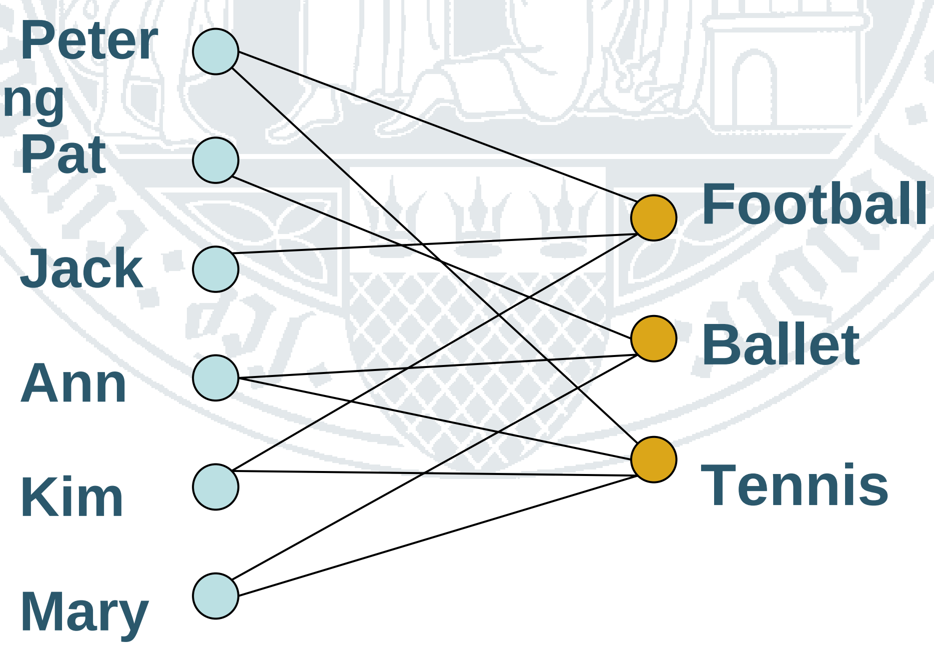
Analyzing Affiliation Networks

Affiliation Matrix, Bipartite Graph and Hypergraph, Rate of Participation, Size of Events

Two-mode network / affiliation network / membership network / hypernetwork

- **nodes can be partitioned in two subsets**
  - **N (for example g persons)**
  - **M (for example h clubs)**
- **depicted in *Bipartite Graph***
- **lines between nodes belonging to different subsets**



Affiliation Matrix, Bipartite Graph and Hypergraph, Rate of Participation, Size of Events

Affiliation Matrix (Incidence Matrix)

- Connections among members of one of the modes based on linkages established through second mode
- g actors, h events
- $A = \{a_{ij}\}$ $(g \times h)$

Event

Actor

	Football	Ballet	Tennis	rate of participation
Peter	1		1	1
Pat		1		1
Jack	1			1
Ann		1	1	2
Kim	1		1	2
Mary		1	1	2
size of event	3	3	4	

Affiliation Matrix, Bipartite Graph and Hypergraph, Rate of Participation, Size of Events

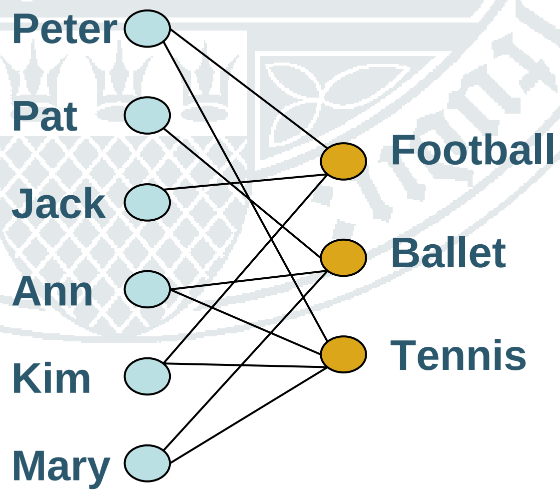
- Sociomatrix [(g+h) $\times$ (g+h)]

	Peter	Pat	Jack	Ann	Kim	Mary	Football	Ballet	Tennis
Peter	-	0	0	0	0	0	1	0	1
Pat	0	-	0	0	0	0	0	1	0
Jack	0	0	-	0	0	0	1	0	0
Ann	0	0	0	-	0	0	0	1	1
Kim	0	0	0	0	-	0	1	0	1
Mary	0	0	0	0	0	-	0	1	1
Football	1	0	1	0	1	0	-	0	0
Ballet	0	1	0	1	0	1	0	-	0
Tennis	1	0	0	1	1	1	0	0	-

Affiliation Matrix, Bipartite Graph and Hypergraph, Rate of Participation, Size of Events

- ***Homogenous*** **pairs and** ***heterogenous*** **pairs**
- $X_r^N (g \times g)$, $X_r^M (h \times h)$, $X_r^{N,M} (g \times h)$, $X_r^{N,M} (h \times g)$
  - **One-mode sociomatrices X^N [and X^M]**
    - rows, colums: actors [events];
    - x_{ij}: *co-membership* [number of actors in both events] (main diagonal meaningful, e.g. total events attended by an actor)

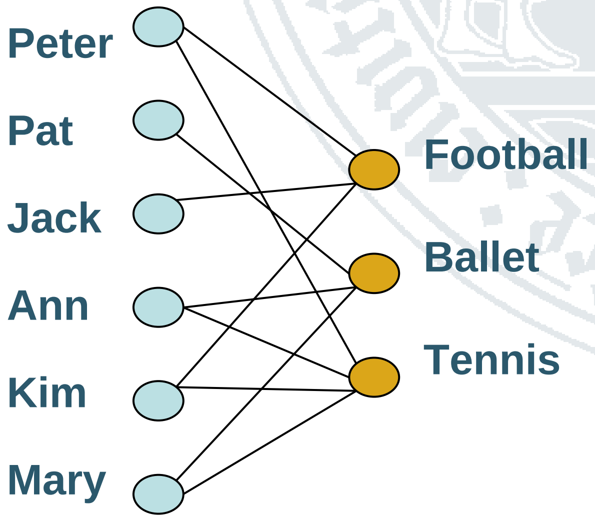
	Peter	Pat	Jack	Ann	Kim	Mary
Peter	2	0	1	1	2	1
Pat	0	1	0	1	0	1
Jack	1	0	1	0	1	0
Ann	1	0	0	2	1	1
Kim	2	0	1	1	2	1
Mary	1	0	0	1	1	2



Affiliation Matrix, Bipartite Graph and Hypergraph, Rate of Participation, Size of Events

- ***Event Overlap / Interlocking Matrix***

	Football	Ballet	Tennis
Football	3	0	2
Ballet	0	3	1
Tennis	2	1	4



Cohesive Subsets of Actors or Events

- ***clique at level c (cf. also k-plexes, n-cliques etc.)***
  - **subgraph in which all pairs of events share at least c members**
- ***connected at level q***
  - **subset in which all actors in the path are co-members of at least q+1 events**

When is which Centrality Measure Appropriate?

Source: *Borgatti, Stephen P. (2005) Centrality and Network Flow, Social Networks 27, p. 55-71*

Assumptions of Centrality Measures

- Which things flow through a network and how do they flow?

	Transfer	Serial	Parallel
Walks	Money exchange	Emotional support	Attitude influencing
Trails	Used Book	Gossip	E-mail broadcast
Paths	Mooch	Viral infection	Internet name-server
Geodesics	Package Delivery	Mitotic reproduction	<no process>

Assumptions of Centrality Measures

- **Example: Betweenness Centrality**
  - Information travels along the shortest route
  - Probability of all geodesics being chosen is equal

	Transfer	Serial	Parallel
Walks	Random Walk Betweenness	?	Closeness Degree Eigenvector
Trails	?	?	Closeness Degree
Paths	?	?	Closeness Degree
Geodesics	Closeness Betweenness	Closeness	?

Adequacy of Centrality Measures

	Transfer	Serial	Parallel
Walks	Money exchange	Emotional support	Attitude influencing
Trails	Used Book	Gossip	E-mail broadcast
Paths	Mooch	Viral infection	Internet name-server
Geodesics	Package Delivery	Mitotic reproduction	<no process>

Source: *Borgatti, Stephen P. (2005) Centrality and Network Flow, Social Networks 27, p. 55-71*

How to Calculate Geodesic Distance Matrices?

From Adjacency Matrices to (Geodesic) Distance Matrices I – (Reachability)

Repetition: Matrix Multiplication

- **XY = Z**

X

3	0	2
1	4	2

$g \times h$

Y

2	3
1	4
4	2

$h \times k$

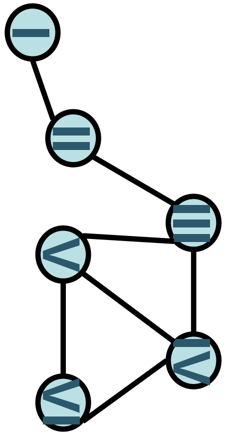
Z

14	13
14	23

$g \times k$

- $z_{ij} = \sum_{n=1}^{h} x_{in} y_{nj}$

From Adjacency Matrices to (Geodesic) Distance Matrices II – (Reachability)



	I	II	III	IV	V	VI
I	0	1	0	0	0	0
II	1	0	1	0	0	0
III	0	1	0	1	1	0
IV	0	0	1	0	1	1
V	0	0	1	1	0	1
VI	0	0	0	1	1	0

X

	I	II	III	IV	V	VI
I	0	1	0	0	0	0
II	1	0	1	0	0	0
III	0	1	0	1	1	0
IV	0	0	1	0	1	1
V	0	0	1	1	0	1
VI	0	0	0	1	1	0

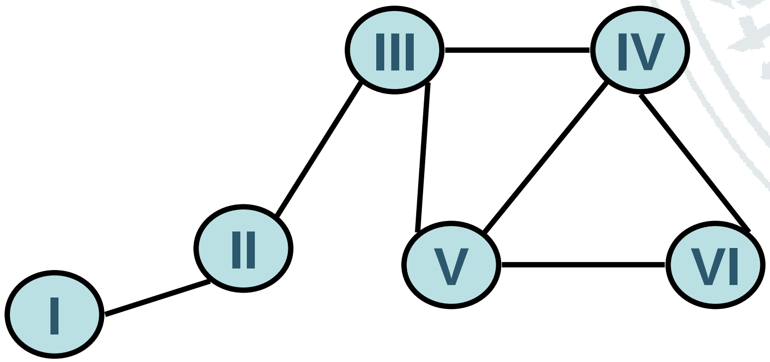
Power Matrix: Multiplying adjacency matrices

- $x_{ik}x_{kj}$ =1 only if lines (n_i,n_k) and (n_k,n_j) are present, i.e. $X^{[2,3,4]}$ counts the number of walks ($n_i n_k n_j$) of length 1 [2,3,4] between nodes n_i and n_j

	I	II	III	IV	V	VI
I	1	0	1	0	0	0
II	0	2	0	1	1	0
III	1	0	3	1	1	2
IV	0	1	1	3	2	1
V	0	1	1	2	3	1
VI	0	0	2	1	1	2

From Adjacency Matrices to (Geodesic) Distance Matrices II – (Reachability)

- $x_{ij}>0$?
  ➔ two nodes can be connected by paths of length ≤ (g-1)
- Calculate $X^{[\Sigma]} = X + X^2 + X^3 + \ldots + X^{g-1}$
- $X^{[\Sigma]}$ shows total number of walks from n_i to n_j

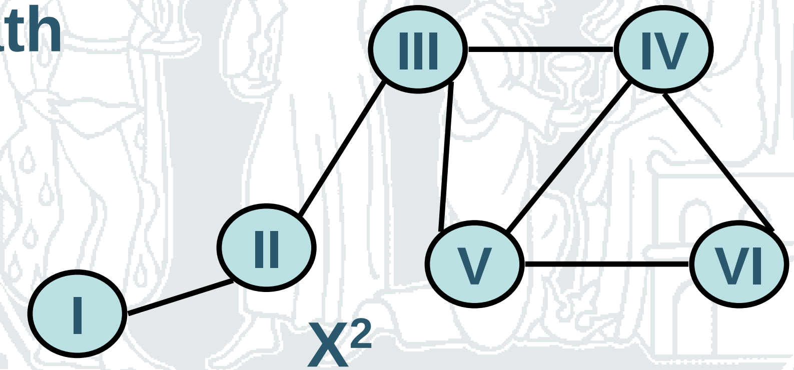


X^2

	I	II	III	IV	V	VI
I	1	0	1	0	0	0
II	0	2	0	1	1	0
III	1	0	3	1	1	2
IV	0	1	1	3	2	1
V	0	1	1	2	3	1
VI	0	0	2	1	1	2

From Graphs to (Geodesic Distance)-Matrices (Reachability) – Geodesic Distance

- observer power matrices
- first power p for which the (i,j) element is non-zero gives the shortest path
- $d(i,j) = \min_p x_{ij}^{[p]} > 0$



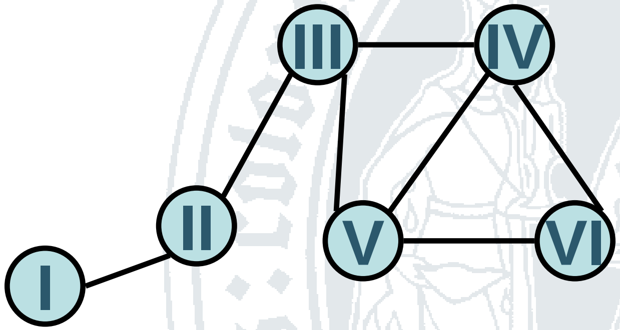
X

	I	II	III	IV	V	VI
I	0	1	0	0	0	0
II	1	0	1	0	0	0
III	0	1	0	1	1	0
IV	0	0	1	0	1	1
V	0	0	1	1	0	1
VI	0	0	0	1	1	0

X^2

	I	II	III	IV	V	VI
I	1	0	1	0	0	0
II	0	2	0	1	1	0
III	1	0	3	1	1	2
IV	0	1	1	3	2	1
V	0	1	1	2	3	1
VI	0	0	2	1	1	2

From Graphs to (Geodesic Distance)-Matrices (Reachability) – Geodesic Distance



Binary, undirected

	I	II	III	IV	V	VI
I	-	1	2	3	3	4
II	1	-	1	2	2	3
III	2	1	-	1	1	2
IV	3	2	1	-	1	1
V	3	2	1	1	-	1
VI	4	3	2	1	1	-

Modelling the Co-Evolution of Network and Behavior

What is Science?

- **What is Science?**
  - **Proving Causalities**

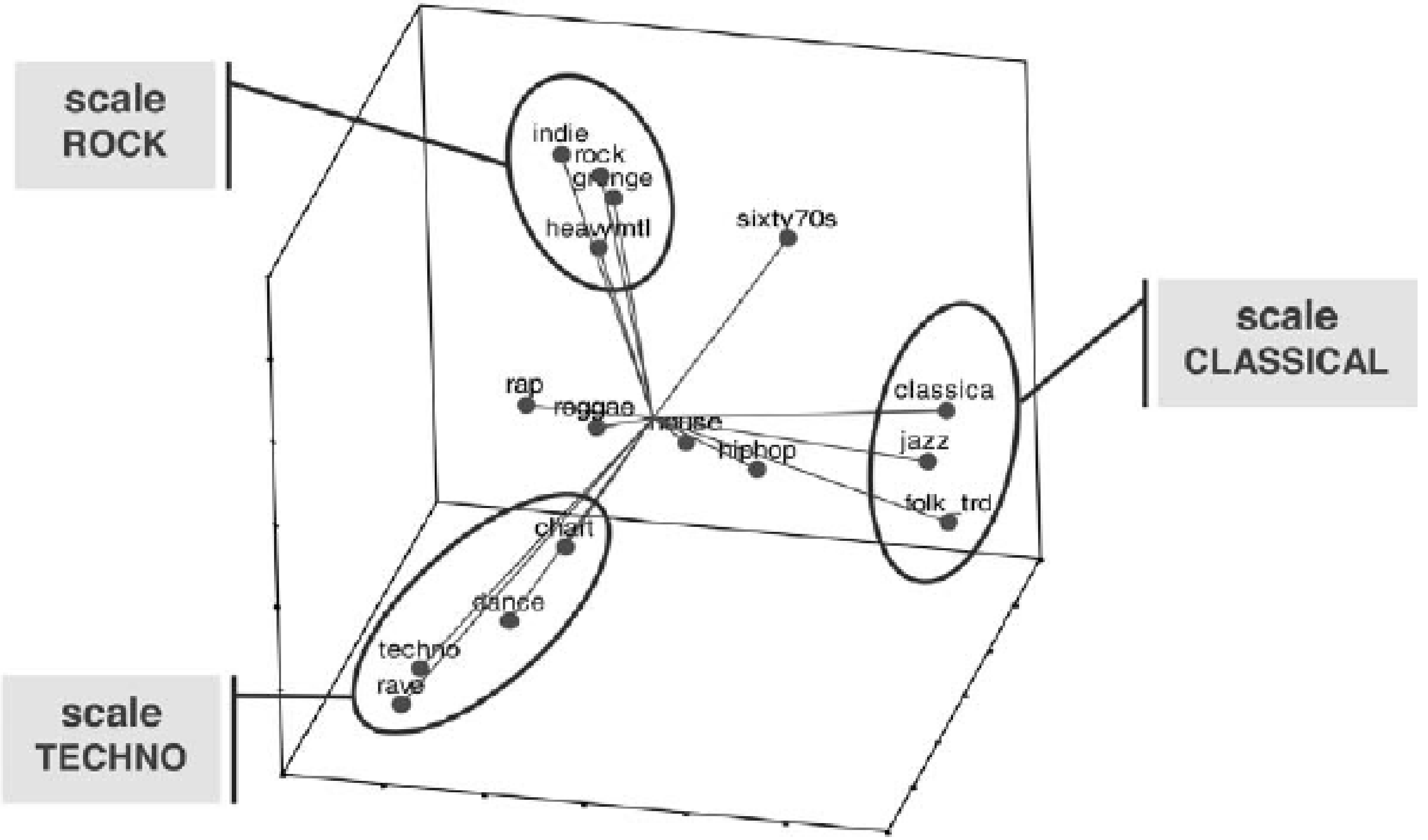
Independent Variable → **Dependent Variable**

- ***Example:***
  - ***Social Capital*: network determines outcomes**
  - ***Homophily*: actor attributes determine network**
- **Endogeneity problem of SNA-studies**
  - **network can be dependent and independent variable at the same time**

➔ Advanced statistical methodologies: Modelling the co-evolution of network and behaviour (…to be cont.)

Example 3) Co-Evolution of Network and Behavior

- study by Steglich, Snijders and West (2006)
- 160 students of a school cohort in Glasgow (Scotland)
- three waves at intervals of one year, starting in 1995 (when students were 13 years old) ending in 1997 (when students were 15 years olds)
- friendship
- demographic variables; music taste, self-reported smoke and alcohol consumption


scale
ROCK
scale
CLASSICAL
scale
TECHNO
indie
rock
grunge
heavymtl
sixty70s
rap
reggae
house
hiphop
classica
jazz
folk_trd
chart
dance
techno
rave

Example 3) Co-Evolution of Network and Behavior – Results I

Submodel	Parameter	Estimate	SE	p value
Network	*outdegree*	−1.89	0.29	< 0.001
	reciprocity	2.34	0.12	< 0.001
	distance-2	−1.09	0.07	< 0.001
	gender homophily	0.80	0.12	< 0.001
	gender ego	0.24	0.11	0.030
	gender alter	−0.21	0.12	0.083
	techno homophily	0.08	0.33	0.798
	techno ego	−0.10	0.05	0.053
	techno alter	0.07	0.05	0.194
	rock homophily	0.11	0.41	0.791
	rock ego	−0.07	0.08	0.357
	rock alter	0.19	0.07	0.006
	classical homophily	1.44	0.69	0.039
	classical ego	0.40	0.17	0.015
	classical alter	0.15	0.17	0.362
	alcohol homophily	0.83	0.27	0.002
	alcohol ego	−0.03	0.03	0.397
	alcohol alter	−0.03	0.04	0.456
	rate period 1	12.45	1.54	< 0.001
	rate period 2	9.56	1.08	< 0.001

Example 3) Co-Evolution of Network and Behavior – Results II

Techno	tendency	0.01	0.25	0.960
	assimilation	0.45	0.18	0.014
	gender	0.25	0.12	0.035
	rock	−0.34	0.10	< 0.001
	classical	−0.13	0.23	0.577
	alcohol	0.07	0.10	0.500
	rate period 1	3.40	0.79	< 0.001
	rate period 2	3.46	0.78	< 0.001
Rock	*tendency*	0.59	0.25	0.016
	assimilation	0.63	0.28	0.024
	gender	0.01	0.19	0.966
	techno	−0.25	0.09	0.003
	classical	−0.34	0.30	0.260
	alcohol	0.11	0.07	0.116
	rate period 1	2.04	0.42	< 0.001
	rate period 2	2.24	0.47	< 0.001

Classical	tendency	0.67	1.30	0.606
	assimilation	0.42	1.17	0.716
	gender	1.57	0.83	0.057
	techno	−0.46	0.40	0.250
	rock	0.64	0.39	0.106
	alcohol	−1.03	0.34	0.002
	rate period 1	0.63	0.38	0.096
	rate period 2	1.43	0.55	0.010
Alcohol	tendency	−0.30	0.37	0.420
	assimilation	0.94	0.27	< 0.001
	gender	−0.06	0.19	0.745
	techno	0.23	0.16	0.145
	rock	0.16	0.16	0.318
	classical	−0.59	0.32	0.067
	rate period 1	1.54	0.36	< 0.001
	rate period 2	2.50	0.54	< 0.001

		Alter		
		Techno	**Rock**	**Classical**
Ego	**Techno**	−0.06	0.25	−1.39
	Rock	−0.15	0.54	−1.31
	Classical	0.02	0.50	1.73

		Impact on odds		
		Techno	**Rock**	**Classical**
Increase	**Techno**	—	−40%	−60%
	Rock	−50%	—	+256%
	Classical	+29%	−49%	—